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THE MATHEMATICS TEACHER

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Future Supply of Science and Mathematics Students[1]

By Dael Wolfle

Commission on Human Resources and Advanced Training, Washington, D. C.

THE TECHNOLOGICAL advances of this country during the Twentieth Century have been made possible by a steadily increasing number of men and women engaged in engineering, the sciences, and mathematics. Without a substantial number of trained and competent workers in these fields we could not have brought the antibiotics, television, radar, the jet engine, and countless other products and processes to their high state of development. While the supply of scientists and engineers has made possible these products of research and development, those products have, in turn, created increasing demands for more scientists, more engineers, more mathematicians, and more technicians. We might go even further in exploring the relationships between technological progress on the one hand and the supply of and demand for scientists on the other. If we had adequate methods of measuring the variables involved, I suspect that we would find some kind of direct relationship between the rate at which we are experiencing technological change and the size of the resulting demand for scientists and technologists.

Certainly we are in a period of rapid change. Some of the military weapons of World War II are now antiques. Some of today's weapons will be just as antiquated five years from now. Medicinal agents which were in their exploratory stages a few years ago are now in daily and economical use. The television industry has mushroomed from almost nothing to its current size in the years since World War II came to an end. There is no need to multiply examples, but I cite the speed of change as an indication that we can anticipate a continued demand. We do not have the techniques with which to state with confidence that next year we will need so many thousand engineers or the year after so many thousand physicists. Exactly how many will be demanded we have no way of forecasting, but that we are in a state of rapid technological and social change and that that condition is likely to continue for some time into the future is a more fundamental fact than any specific estimate of exactly how many scientists and engineers we need at the moment. The number required will continue to

[1] This paper constituted part of the introduction to a conference on *Identifying High School Students with Potential for Science and Mathematics and Providing Opportunities for their Development* which was held in Washington, D. C. on November 13–15, 1952 under the joint auspices of the United States Office of Education and the Cooperative Committee on the Teaching of Science of the American Association for the Advancement of Science. The paper is also appearing in *The Science Teacher*.

increase until the nation's leaders decide, as a result of changed world conditions, as a result of legislative action, or for some similar reason, that the United States is going to slow down its rate of technological advance.

It is widely assumed that the demand for scientific manpower is likely to be greater than will be the supply which can normally be expected, and that it behooves us to attempt to get more bright youngsters interested in science and mathematics in order to augment the future supply. Before examining the probable future supply and considering methods by which it might be increased it seems desirable to take a look at the present supply.

The easier group to describe is engineers, for there is a little greater agreement over who is an engineer than there is over who is a scientist, and there is more information available about the engineering profession than there is about natural scientists as a group.

As of today, the number of engineers in the United States is just about half a million. That is approximately 12 times as many as we had in 1900 and almost twice as many as in 1940. Not all of the engineers have graduated from college, but somewhat over 80 per cent are college graduates and the proportion is steadily increasing. We must expect the great bulk of replacements and of new additions to come from the nation's engineering colleges.

There are too many unknowns involved to permit us to estimate the future supply with precision, but I can indicate the order of magnitude of some of the major variables. For the next few years we can compute from the age distribution of living engineers and from standard life tables that approximately 7,000 of those under the age of 65 will die each year. From college enrollment information we can be pretty certain that the number of new graudates will remain under 25,000 a year for the next several years. Just how many will withdraw from engineering to enter other fields will depend upon future conditions, but the number is always substantial. Moreover, not all engineering graduates go into engineering work, and neither the military services nor their own inclination lead us to expect that all future graduates will become engineers. In short, it is completely certain that engineering can not increase as rapidly during the next few years as it has during the past three or four. There just aren't enough in the pipeline.

As for scientists, the Bureau of Labor Statistics estimated that we had 175,000 employed in 1950. The Research and Development Board has estimated that we have 204,000 in 1952. Who qualifies for the label of scientist is not a question to which there is any one clear answer. Preliminary data suggest that in the neighborhood of 125,000 college graduates enumerated in the 1950 census were classified as scientists. That figure is almost exactly the same as an estimate made by the Research and Development Board that there were 123,000 research scientists in the United States in 1950. Their estimate of research scientists for 1952 is 135,000. Perhaps we ought not to take any of these specific figures too seriously. We can, however, take from them the generalization that the country's scientists are less than half as numerous as the country's engineers. From comparable estimates for earlier years we can also make the generalization that the nation's scientists have approximately doubled in number since 1940.

A study conducted by the American Council on Education for the Office of Naval Research has provided estimates of the number of living persons who have received Ph.D degrees in the natural sciences from American universities. The estimate is that there were 39,000 such persons alive and not over 70 years of age in 1950. When we add the more recent Ph.D.'s in the sciences we arrive at a current estimate of 46,600 for the number of

persons with Ph.D. degrees from American universities who are living and under the age of 70.[2]

There is an interesting property of these estimates; engineers, scientists, research scientists, and persons holding the doctorate in science have all, in round numbers, doubled since 1940. During the war years the increase was small; most of it has taken place since 1946. During those years we have made great strides in increasing our resources of scientific manpower. But the United States has not been alone in that regard. A paper published recently by Shimkin of the Russian Research Center at Harvard[3] indicates that graduates of higher educational institutions employed in professional positions in the USSR more than doubled in number between 1937 and 1952. Russia too has been making great strides in increasing her corps of engineers, natural scientists, agricultural specialists, physicians, industrial leaders, language specialists, and other professional workers. We can certainly not be complacent that our rapid increases insure us continuing scientific superiority.

We should therefore analyze school and college populations to see what quantity and quality of replacements and additions to our scientific population we are likely to secure during the next few years. Of the general result we can be certain; we are not going to have as many graduates in the sciences during these next few years as we have recently had. The GI wave is about over. For the next few years we will have to depend upon the normal age group for our college graduates. And current age groups are small. The number of people reaching college entrance age is now at the bottom of a trough, smaller than it was during the Forties and much smaller than it will be when the large baby crops of the war and postwar years grow old enough to enter college. Out of the current small age groups, however, we can expect a slightly but steadily increasing percentage to graduate from college. Our studies of the past trends lead us to the conclusion that the number of college graduates, expressed as a percentage of all young men and women reaching the age of 22, increases on the average by .3 of a percentage point each year. In terms of concrete numbers, that means that we can expect 265,000 graduates in 1955 and 326,000 in 1960. These figures can be compared with the 186,500 we had in 1940, the greatly inflated 434,000 in 1950, and the less inflated 325,000 in 1952. Since the peak year of 1950, graduating classes have been getting smaller and smaller. We can expect the decrease to continue until about 1955. The number will then begin to increase again.

How many of the future graduates will have specialized in the sciences is a question of particular interest. We have attempted to determine the past trends with respect to the distribution of college graduates among different fields of specialization. In order to do that we have combined partial data from a number of sources and have supplemented those data with a couple of historical sample studies of our own. The net result is a conclusion that the percentage of college graduates who have majored in one of the natural sciences has been dropping rather steadily from 1900 to 1950. In 1900 a fourth of all recipients of bachelor's degrees had specialized in one of the sciences; in 1950 only an eighth had majored in a science. During the same 50 years the percentage majoring in agriculture and engineering has remained fairly constant. The percentage for engineering has been dropping slightly for the past five years, but will begin to show an increase in 1955. The number of freshman engineering students in the fall of 1951 was some ten per cent higher than it had been a year earlier, and in 1952 freshman engineers

[2] Douglas E. Scates, Bernard C. Murdoch, and Alice V. Yeomans, *The Production of Doctorates in the Sciences: 1936–1948* (Washington, D. C.: American Council on Education, 1951).

[3] Demetri Shimkin, "Scientific Personnel in the USSR," *Science*, CXVI (1952), 512–513.

were up 31 per cent above 1951. Both of these figures show that engineering has for the past two years attracted an unusually large number of freshman men. The graduating class of engineers in 1955 can be expected to be a little above the class of 1954 and the class of 1956 can be expected to show an even larger increase.

Figures are not available for equally precise statements about the number of graduates to be expected in mathematics, physics, chemistry, and the other sciences, but there is a very real possibility that a part of the prospective increases in engineering graduates will be at the expense of decreased numbers in mathematics and the sciences.

While these figures and speculations do not indicate just how many future graduates in the sciences we can count on, they permit several important conclusions: (1) The total number of college graduates will continue to decrease until about 1955, after which the number will begin to rise; (2) The number of engineering graduates will decrease to 1954 and then begin to increase; (3) The number of graduates in mathematics and the natural sciences will drop for the next few years and may continue to decline longer than will the total number or the number in engineering, for a part of engineering's gain may be coming out of the science fields; (4) The graduates in the sciences during these immediately future years will not be numerous enough to continue the rapid rate of increase of recent years.

All of these statements have been made before. They are, in fact, the background against which there have developed a number of attempts to interest more high school graduates in careers in engineering and science. The efforts of the Engineers Joint Council is one example. The Future Scientists of America organization is another. In support of such ventures is clear evidence that there are lots of bright boys and girls who finish high school but do not enter college. If we take as a point of reference the level of ability of the average college graduate of today, as measured by the score he makes on a test of general academic aptitude or intelligence, we get the following figures. Of all of the boys and girls in the country who are at least as bright as the average college graduate—about the top 15 per cent of the total population—forty per cent finish high school but do not go to college; twenty per cent start college but do not finish; and forty per cent become college graduates. If we take a more or a less selective level of ability as our standard of reference the percentages naturally change, but the one illustration is sufficient to indicate that there are lots of able youngsters who might make good college material but who do not get to, or at least not through, college.

The reasons for this loss are multiple. The first to occur to most of us is lack of money. But money is not the only reason holding these boys and girls out of college. In a not yet published study of all of the Minnesota high school graduates of 1950, Ralph Berdie found that about half of those who were in the top ten per cent as measured by the American Council on Education Psychological Examination but who were not going to college the following year said that they would go to college if they had the money. The other half indicated that they would not plan to attend college even if funds were available. The student's attitudes, the educational aspirations which he develops for himself, the attitudes towards higher education of his family and friends, all of the elements which we can lump together under the general heading of motivation and interests appear to be at least as powerful an element in keeping a number of potentially good students out of college as is the lack of money.

I summarized the first portion of this paper with the statement that we can not expect as many college graduates in engineering and the sciences in the next

few years as we have had in the past few years and not enough to enable the scientific fields to continue their growth at the recent rapid rate. Earlier I had pointed to the great technological changes of the current period as evidence of the continued high need. We can now add another generalization: there is a sufficiently large supply of ability in the youth of the country to make it possible to double or perhaps more than double the number of college graduates of the caliber which now constitutes the upper half of college graduating classes.

The importance of attempts to interest more bright youngsters in science and mathematics is therefore established. Science needs good brains. The nation's advancing technology needs a steady stream of new scientifically trained recruits. To identify high school students with potential for science and mathematics and to provide opportunities for their development is a contribution to the talented young people involved and a contribution to the future welfare of the society they can serve.

But the task of identifying high school students with potential for science and mathematics is a problem which involves a number of issues. How, for example, does a high school student with potential for science and mathematics differ from a high school student with potential for medicine, economics, linguistics, or any one of a number of other fields which are also important and which also have a right to claim a portion of the most promising young students. Our own studies of the intellectual quality of students who specialize in different fields indicate that students of mathematics and science are on the average slightly better than the generality of college students but that overlap from one field to another is very large indeed. There are other fields which now command as qualitatively good a group as do the natural sciences. For example students receiving degrees for major work in psychology, English, foreign languages, medicine, and law average as high as those whose baccalaureate majors were in the natural sciences.

A deliberate effort to draw a larger number of abler students into science and mathematics can be developed along either of two lines. If the appeal is general, to all high school students who are potentially good scientists, then the appeal is one which may reduce the number of students entering other fields. If the effort is directed at students who would not otherwise enter college, then science and mathematics may profit without detracting from other fields. This second alternative seems to me both the more desirable and the more difficult of the two alternatives—the more desirable because it is calculated to increase the total number of high quality people receiving advanced training and the more difficult because it is hard to plan a program—be it scholarship, counseling, or what not—which will influence those students who had not otherwise planned to go to college without also influencing the choices made by those who were planning to go to college anyway.

It seems highly desirable to attempt to interest a larger number of the best high school graduates in going to college. They are the potential leaders of tomorrow. With a bachelor's or some higher degree and with the ability which makes their college training profitable they will be the industrial managers, the research scientists, the teachers, humanists, social scientists, military leaders, and others who will manage our complex social, military, industrial, and governmental machinery, and we hope, lead us successfully through the troubled times in which we live.

It is easier to make narrow plans than broad ones. It is easy to say that we should lead more students into the fields of science because the sciences need more competent workers. They do. But I am reminded of the policy statement of the

(Continued on page 240)

General Ways to Identify Students with Scientific and Mathematical Potential

By Howard F. Fehr
Teachers College, Columbia University, New York City

The task of identifying giftedness is not an easy one. In December 1940, a two-day conference and workshop on education for the gifted was held at Teachers College in honor of the great work done by Leta Hollingworth. One section of outstanding teachers and educational research workers devoted itself entirely to the task of identifying the gifted child. The conclusion reached was: "At the present time we have practically no adequate instrument for identifying the gifted."[1] In *The Gifted Child*[2] edited by Paul Witty, we read, "Present means of identifying and guiding the gifted leaves much to be desired," and the rest of the brief chapter gives adequate support to this stand both in its meagerness of discussion and the problems for investigation that are raised. Even the latest book on *Educating Gifted Children*[3] a report on the Hunter College Elementary School Program by Gertrude Hildreth and others, takes the same point of view regarding our ability to detect the gifted at an early age. Formal tests seem to be the one criteria that most people rely upon.

An acceptable definition of giftedness or potential in science and mathematics study should be given at the outset, for successful search for this talent must depend on knowning what we are seeking. Most of the readers are, no doubt, familiar with a number of definitions of giftedness or talent, and no repetition will be made here. For the most part they use the I.Q. as a basis, or a very small fractional part at the top of the school population as measured on some type of aptitude test. Since in this discussion we are concerned with informal or general means of identification, the following informal definition is proposed. "The gifted child is one who shows to an exceptionally high degree, the ability to do work with ideas. He is exceptionally capable in thinking, that is in the manipulation and creation of abstractions of word and number."

At the outset, it should be made clear that we do not subscribe to only this one type of giftedness. Psychologists today are inclined to recognize others, for example mechanical giftedness, and social or political talents, and these may not be related to the type of giftedness under discussion. We need only refer to the possession of space perception abilities. Minds that are able to deal in high abstractions and logical relations frequently fail miserably in space perception tests. Yet this space perception ability (abilities) is an important essential in modern engineering and technological design.

The identification, other than by formal testing, must come about through observation. This calls for judgments on the part of the teacher, as he observes and interviews the child; the parents as they report on the child's behavior in and around the home; and the associates as they report on their relations with the child at play and at work. All studies thus far point out the extremely poor record of identification of the gifted child on the part of teachers. They miss the boat to the extent of selecting only fifteen correctly out of one hundred they name. How many they miss by not naming is unknown. Herbert Carroll in his *Genius in the Making* lists three reasons for this. (1) The personal equation, that is likes

[1] Teachers College Record, February 1941.

[2] Paul Witty, *The Gifted Child* (Boston: D. C. Heath & Company, 1951), p. 12.

[3] Gertrude Hildreth, *Educating Gifted Children* (New York: Harper Brothers, 1952).

and dislikes; confusion of friendliness, obedience, conformity, beauty, and the like for ability and talent in science; (2) lack of standards, that is, the teacher has no valid estimate of memory, curiosity, abstract thinking, with which to compare these qualities in the student's classroom performance; and (3) chronological age factors, that is, the teacher overlooks the difference in ages of two students of equally high grade performance.[4]

There may be other reasons for teachers' inability to select talented youth. They may lack psychological knowledge and bases for making judgments; they may be average in intelligence themselves and hence lack the ability to make the keen discernments, valid introspections, and to supply opportunities for giftedness to express itself. Teachers must be trained to be better observers of essential traits that mark genuine talent. Insofar as teachers are able, they should begin to put the informal means suggested herewith (and others) to work. They should keep anecdotal records of their judgments and follow through for several years the children on whom they make judgments. They can correct errors, improve and strengthen their abilities. They should emulate Leta Hollingworth who was a genius at "introspection." It would appear that the most immediate problem is one of aiding teachers to make better selections than they have in the past.

Characteristics of gifted children have been enumerated by all the researchers in this field. Here are stated the more generally recognized traits and suggested methods of identifying them in the classroom. *The mark of giftedness lies not in the possession of these characteristics, but in possessing them at a high deviation from even the more able students.* Illustrations were chosen from my own and my colleagues' experiences, with the hope that these incidents will help the reader to recall similar situations, and to make some sort of generalization for recognizing the trait.

The most significant trait is an *extraordinary memory.* Gifted children have a mental storage capacity that is truly amazing, and it seems to be in part a result of relational thinking. Perhaps a simple example is the boy who in the senior year of high school could give at sight the square of any number between one and one hundred. He had learned the squares up to twenty-five in the eighth grade, and of his own accord had taken the task of learning all of them up to one hundred. Five years later he could call forth the answer, although in the interim he had very little practice in their use. He could not explain how he remembered these numbers. *The gifted child seems never to forget.*

A second significant trait is the ability to do *abstract thinking at a high level.* Brilliant students make generalizations quickly and accurately.

In a first grade class a boy gave his weight as 60 pounds. The teacher told him to stand on one leg and asked him his weight then. He, and several other children, said 30 pounds. But one girl insisted he still weighed 60 pounds, since standing on one leg had no effect on changing your weight. The important fact is not that these children could take one-half of 60 correctly at age 7, but that the one child could detect the generalization about weight.

A similar generalization is shown by the following student in plane geometry. The class had been given Morley's theorem on the trisectors of the angles of any triangle meeting in points that formed the vertices of an equilateral triangle. Next day, the student told the class he had discovered, that while this theorem about trisectors of angles was true for any triangle, it was not true for polygons of more than three sides, unless the polygon was regular, in which case the intersections were vertices of another regular polygon of the same number of sides. While this appears per-

[4] Herbert A. Carroll, *Genius in the Making* (New York: McGraw-Hill Book Co., 1940).

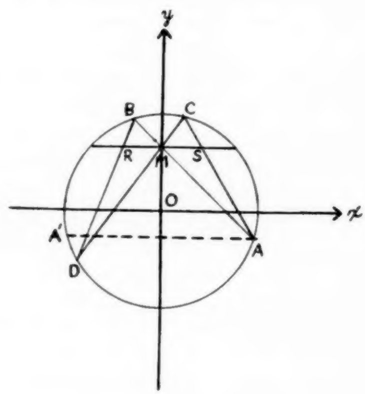
haps a trivial discovery, its significance is seen in the fact that no other high school student or textbook writer appears to have made the generalization, for it appears in none of the textbooks with which the writer is acquainted.

A third trait shown by the gifted is that of *applying their knowledge*—seeing mathematics and science in their environment. They are facile in the application and manipulation of symbolic and relational thinking. An eighth grade boy was reported as non-cooperative and deficient because he refused to do the strong-arm calculations required in his science class. In mathematics he was interested in ideas, but also resisted computational practice. On return from his summer vacation, which was spent in a lumber camp, the boy exhibited a new tape he had devised for measuring diameters of trees. On an ordinary tape he marked 1 at $3\frac{1}{7}$ ft., 2 at $6\frac{2}{7}$ ft., etc., and then divided each of these sections into 100 equal parts. The readings on the new tape gave the diameter of the tree to the nearest hundredth of a foot. This boy readily applied his mathematical ideas to a practical problem. He went on in his studies and is now a brilliant science student in college.

A fourth recognizable trait is *intellectual curiosity*. The gifted child has motivation other than making good grades or conforming to a classroom pattern of instruction. He is speculative, but with it all arrives at his conclusions through a very high sense of logic.

A ninth grade algebra class (above average) had been introduced to exponents and the use of the slide rule. They were given student rules with which to work. One boy asked if a slide rule could be made to the base 5 instead of the base 10. Told that it could, but it would be quite a lot of work, he developed a set of logarithms to the base 5, operating in the base 5 made an A, B, C and D scale to the base 5, and produced a slide rule, probably the first slide rule to the base 5 that was ever made. Later he made a simple computing machine to the base 5. This boy is literally a genius in mathematical analysis, and in his junior year could do the whole year's work before entering the class. He learned nothing new in class because the instructor taught only a fixed course. Fortunately, he had recourse to other mathematics professors and was well through calculus as he entered his senior year in high school.

A fifth trait is the *persistent goal-directed behavior*. Children with this trait have the capacity for long and deep concentration on their problems. They do not give up.

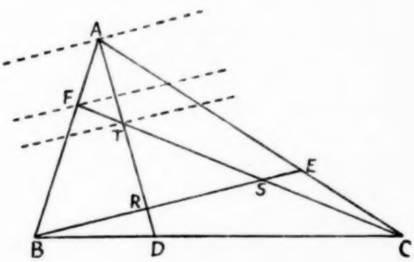


A good student in plane geometry solved the "Butterfly" original after hours and hours of concentration. This original concerns the fact that if through the midpoint M of a chord, any two chords AB and CD are drawn, then AC and DB meet the chord at S and R with $RM = MS$. The teacher had not found a solution by Euclidean methods and so resorted to analytic geometry to establish the theorem. The student refused the easy way and finally discovered the proof by drawing a parallel to the chord RS through A and showing A', R, M and D to be concyclic.

This stick-to-itiveness is a rare trait and is not to be confused with that of the plodding individual who tries and tries and finally muddles through to a solution. The teacher must distinguish between the

two. The gifted student is never satisfied with muddling through. He refines and polishes, and even seeks better solutions.

A sixth characteristic, hard to define, may be given the title *intuition*. The talented student has *insight* and *penetrability* to the problem under consideration. He is mentally quick; he sometimes grasps the solution in a manner that is almost incomprehensible to teachers. His solutions are amazing.



An example of this is the response of a student to the well-known Nedian theorem, that the solution could be obtained by drawing parallels to any Nedian through the remaining points. When pressed as to why he selected these lines, his response was that it was plain that it was a problem in proportionality. His attack was correct.

The story of Gauss on the sum of an arithmetic progression is another illustration of this intuitive grasp of solutions. I may add that the biographies of great mathematicians and scientists give many clues to giftedness.

A seventh characteristic is the *high vocabulary and facility of expression* in scientific matters. If teachers will allow students to work on a project or paper and then read these papers, they can readily detect high ability as contrasted with ordinary ability. I need only cite the papers presented in the "Talent Search" contests as prime examples. In fact the scientific language and lucidity of expression on these papers can overwhelm the teacher.

An eighth characteristic, and one that is very significant, is the presence of a *hobby which is hard ridden*. The gifted student usually has a special interest, in which he reads and works, and he constantly relates it to everything that goes on in the class. Take the case of Libby.

In geometry, during the study of constructions, she made all kinds of beautiful designs, with color. She enjoyed curve stitching and invented some of her own designs. The teacher cautioned Libby about wasting time on inconsequentials rather than improving her ability in logical analysis. When the study of locus took place, Libby located every conceivable object in a room equally distant, or at a fixed distance, from designated places or paths in the room. When a unit in coordinate geometry was undertaken, Libby coordinated every community institution and every pipe and sewer line with respect to two main streets in the commercial, and then in the residential area of her community. Today Libby is a town planner of national reputation in a sizable city. The teacher could have helped by encouraging the hobby and directing it, rather than pushing the regular course material.

The interest may be in soil, plants, animals, electronics, or mathematics, but it is always there and always crops out. Students who possess such an interest are not wasting time at it.

A ninth characteristic is *virtuosity*. The gifted student gives things a new turn. It is not only curiosity, but it is a creative aspect that even able students do not seem to possess. This was evident in the case of Abel who was attempting to find an algebraic solution to a fifth degree equation. Not succeeding, he gave the problem a new twist, by an idea that was altogether new in mathematics. He probably said to himself "Maybe I can prove that the solution is impossible." Another illustration is the case of a student in the senior year in high school studying conic sections. The cone was cut by planes giving the usual sections. This student raised the question, what happens when you cut the cone by a

curved surface, for example, a parabolic beam of light. He devised a rotating conical surface, threw a parabolic beam on the cone and investigated the resulting space curves. This same student also invented (in his senior year in high school) a machine for demonstrating the laws of multiplication for quaternions. He is now a researcher in the field of electronic computers and differential analyzers for the U. S. Air Force. The students who ask intelligent questions that state problems in new and more general forms are probably the creators of tomorrow.

In connection with traits, there is an excellent article by Zachariah Subarsky in the May 1948 issue of the *Scientific Monthly*. In it the following characteristics are noted and well illustrated: (a) innate curiosity, (b) observation of discrepancies in a given situation, (c) the ability to formulate hypotheses for testing, (d) the ability to translate observational facts into quantitative or mathematical terms, and (e) the engineering mindedness which creates and sets up equipment and laboratory procedures for testing hypotheses.[5]

A final trait I would mention, because of its real significance in selection of gifted students, is their *sound knowledge of advanced areas* in the field of mathematics and science. They have read and studied by themselves and turned up in class with neat solutions of problems in terms of calculus, analytic geometry, or even differential equations, while the ordinary algebraic solutions are long and involved. The evidence of such knowledge must certainly have been detected in their own experiences by many of the readers of this article.

A word about teaching and teachers in relation to observing traits that signify giftedness in science is in order. The teaching must permit opportunity for experience and exploration in various mathematical and scientific areas. Unless all students are given an opportunity to display their talents, it will be impossible to unearth the exceptional talent and characteristics that have been enumerated. This means that the teacher who teaches a textbook only, with definite assignments for all and nothing beyond these assignments, will hardly be in a position to discover talent. Conformity to a set pattern of instruction, doing the teacher's assignment perfectly, cooperating and satisfying, as fine qualities as they may be, are not signs of giftedness, and teachers should know this. Latent ability may crop out at a most unexpected time, and unless opportunity is continuously given for exploration, for experiencing, for creativeness, and for generalizing within the teaching of our science, we can hardly hope for the characteristics of giftedness to be displayed.

To summarize, some important characteristics of giftedness in science are, extraordinary memory, high ability in abstract thinking, ability to apply knowledge, intellectual curiosity, persistent goal-directed behavior, intuition, insight, and penetrability in a problem, high vocabulary and facility in verbal expression, a hobby which is all-absorbing, originality or virtuosity, and advanced scientific knowledge.

[5] Zachariah Subarsky, "What is Science Talent?" *Scientific Monthly*, LXVI (May 1948), 377–82.

The readability boys and their word-counting machines have gone too far, according to Stephen E. Fitzgerald in a *Saturday Review of Literature* article (February 14, 1953) entitled "Literature by Slide Rule." Mr. Fitzgerald says he believes that research in readability has had a wholesome effect on some writers and their writing. But from a science, readability has become an industry. Formulas, counting devices, straight-edge gimmicks—all seek to put a strait jacket on writing. In addition to the readability techniques, writers must also have something important to say; they must assume that their readers have enough intelligence to "catch" what the writer is trying to "pitch"; and they must remember that it is ideas and not words alone that count.

The Organization of Instruction in Arithmetic and Basic Mathematics in Selected Secondary Schools

By LEE IRVIN

Special Research Project Director, Southern Section California Mathematics Council, Rosemead, California

DURING the school-year of 1951–52, the Southern Section of the California Mathematics Council sponsored a research project concerned with the offerings in arithmetic and other non-traditional mathematics courses at the high school level.[1] The purpose of the project was two-fold: (1) to discover current practices which might be adapted for use in improving non-traditional mathematics courses at the secondary level and (2) to find bases for making recommendations for the reorganization of the high school mathematics program. The nature of the study, therefore, was that of "action research" such as was recently proposed by the Commission on Life Adjustment Education.

Representatives of ninety-two schools, situated in thirty-five states and the District of Columbia, took part in the project by supplying information concerning their present offerings and by outlining certain plans in progress for improving their programs. The schools were located by the following means: (1) through references in the literature to experimental courses or successful procedures in progress at the time, (2) upon the advice of certain city and county supervisors, (3) by contacts made with certain leaders in the field of mathematics education, (4) upon the advice of certain curriculum research specialists, and (5) through other reports from the field.

The reports included information regarding three main aspects of the problems involved in the reorganization of the mathematics program: (1) the administrative aspects of curriculum reorganization, (2) the selection and organization of subject matter content and activities for the non-traditional mathematics courses, and (3) teaching procedures and methods of evaluation.

Ten specialists, five in the area of mathematics education and the others in the area of curriculum research and reorganization, cooperated in the project by offering useful information and suggestions or by assisting in the evaluation of the findings. The cooperating specialists were as follows:

A. Mathematics education specialists—
   - DALE CARPENTER, Mathematics Education Supervisor, Secondary Curriculum Division, Los Angeles City Schools
   - WILLIAM A. GAGER, Professor of Mathematics, University of Florida, Gainesville, Florida
   - DONOVAN A. JOHNSON, Head of the Mathematics Department, University High School, University of Minnesota, Minneapolis, Minnesota
   - F. LYNWOOD WREN, Chairman of the Mathematics Department, George Peabody College for Teachers, Nashville, Tennessee
   - JAMES H. ZANT, Professor of Mathematics, Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma

B. Specialists in curriculum research or reorganization—
   - HAROLD ALBERTY, Professor of Education, Ohio State University, Columbus, Ohio
   - HARL R. DOUGLASS, Director of the College of Education, University of Colorado, Boulder, Colorado
   - JOHN R. EALES, Curriculum Coordinator, Division of Secondary Education, Los Angeles County, and Director of the South-

[1] For the purposes of this report, the term *non-traditional mathematics courses* will refer to all high school mathematics courses other than the traditional college preparatory sequence of Algebra I, plane geometry, Algebra II, trigonometry, and solid geometry.

ern Section of the California Mathematics Council

FREDERICK J. WEERSING, Curriculum Consultant and Professor of Secondary and Higher Education, University of Southern California

J. WAYNE WRIGHTSTONE, Director of Educational Research, Board of Education of the City of New York

The opinions of the specialists listed above were used constantly in classifying and analyzing questionnaire replies, as well as in making the concluding recommendations regarding the reorganization of the high school mathematics program.

The questionnaire constructed for use in the investigation of the selected schools included fifteen items for obtaining general information, twenty-five regarding curriculum reorganization practices, thirty-seven on teacher preparation and related matters, and thirty-five concerned with classwork and methods of evaluation. Information secured by the questionnaire was supplemented by classroom visits to ten of the schools in question, by a library survey of related literature, and by a more detailed study of the work being offered by fifteen of the selected schools, each of which gave one or more of nineteen courses that seem to be representative of current non-traditional mathematics offerings.

SUMMARY OF FINDINGS

It was found that seventy-four of the ninety-two cooperating schools offered either a three-track or a multiple-track mathematics program; sixteen offered a double-track program, either at the ninth or twelfth grade level or at the ninth and twelfth grade levels; twenty-nine offered related mathematics in connection with industrial arts and agricultural curricula; and nine of the schools offered special-interest mathematics courses for girls in home-making, household arts, or pre-nursing curricula.

Although several of the non-traditional mathematics courses offered were of the "fused" or "unified" type and many were of the "consumer mathematics" type, the majority were courses in arithmetic of varying degrees of difficulty, stressing understandings of number relationships and the basic principles of mathematics together with the uses or applications of such knowledge.

Forty-three of the schools included in the selected group offered their non-traditional mathematics courses in a two-to-four year sequence, and thirty-three schools offered two or three differentiated non-traditional courses. The cooperating specialists, for the most part, agreed that a carefully developed sequence of two to four years would probably afford the best means of attaining the objectives of the non-traditional mathematics courses, and they further recommended that basic concepts and principles be taught through a spiral treatment of mathematical topics.

Although twenty schools (22 per cent) offered related mathematics instruction in classes of a modified core section type, the investigation revealed little use of the core curriculum at the high school level. Only four of the cooperating schools (4.4 per cent), two of them in California, reported the use of such an organizational plan.

Course outlines provided by the cooperating schools reveal that steady progress is being made toward meeting the students' needs through the careful selection and organization of subject-matter content as well as through the use of a wide variety of modern supplementary materials. The large number of non-traditional courses being offered throughout the nation in the area of mathematics may or may not be encouraging from the viewpoint of the curriculum specialist. One of the contributing curriculum specialists, however, made the following comments:

> In an ideal school, the mathematics needed by all for general citizenship education would be included in the core in a block of time devoted to major problems of living, and skills would be acquired in a social context. No mathematics would then be required outside of the core, either at the junior or senior high school level.
>
> Outside the core, beginning with the ninth grade, I would provide a rich elective offering of mathematics tailored to meet the specialized

needs of the students—intellectual, aesthetic, and practical.

Despite the fact that the Commission on Post-War Plans of the National Council of Teachers of Mathematics has recommended a double-track mathematics program, the specialists in mathematics education who cooperated in the project are now agreed that the modern high school should offer at least a three-track mathematics program, and one of the specialists recommends that a multiple-track program be extended into the special-interest areas. All were generally agreed that business arithmetic and consumer mathematics, either as separate courses or as units in some sequential course, should be offered at the upper high school level. Many of the applications which should be included in such a study could thus be presented to students who would be of sufficient maturity to understand and appreciate them. Both the mathematics education specialists and the curriculum specialists agreed that there should be some program whereby "maintenance of the fundamental skills" or "an increased ability to use mathematical principles" would be assured to whatever extent possible, and two of the specialists recommended that "more consideration be given to mathematics in the core curriculum."

Gager, in a discussion of the problem of special-interest mathematics courses, wrote as follows:

> I see very little use in most of our secondary schools for specialized mathematics courses. For instance, more than half of the students who might take a course in the mathematics of farm management or in agricultural problems would, in all probability, never follow agricultural work of any kind. It seems to me that we need to get the basic ideas and principles across to our students, and the special fields could then be mastered as the need for any particular type of mathematics developed.

Although he has previously been inclined to go along with the Commission on Post-War Plans in the recommendation of a double-track mathematics program, Gager now has the following to say in regard to this matter:

> In general, I have not been in favor of a third track in the mathematics program. However, due primarily to social promotions in some school systems and possibly to other factors, there are some students who come to the ninth grade with what appears to be a total lack of mathematical ability. I feel that these students should not be allowed to ruin the opportunities of those who are ready to progress successfully in functional mathematics; therefore, I believe that such students must be provided with special remedial or "refresher" courses.

Zant, another of the cooperating specialists, wrote as follows concerning the recommendation of a double-track mathematics program:

> I am now convinced that we need a three-track program: one for college preparation [of science, mathematics, and engineering majors], one for the low group, and one for the relatively large middle group who may or may not go to college but who will carry on the real work of the community in the future.
>
> These students will have need of a broad, basic block of mathematics that is not covered by shop mathematics, by consumer mathematics, or by commercial arithmetic. They should be offered a strong sequential course planned for students with average or more than average ability who will have no need for the specialized traditional mathematics courses.

Wren put the matter in question form and, at the same time, pointed out one of the most serious obstacles to be surmounted in the formation of good non-traditional mathematics courses:

> Why is it that we, who work in a field of thought which emphasizes the attitudes and procedures of critical thinking, should be inclined to postulate a pattern of subject-matter organization whose major cause for existence may be mere tradition? . . . One of the major obstacles encountered in trying to give dignity to [the non-traditional mathematics courses] is the inability to get the approval of our fellow workers in the field of mathematics. . . . We need to re-think the entire mathematics program from the primary grades through junior college. . . . We need more work of this kind on a national scale.

It seems that the majority of the cooperating specialists evidently believe that the recommendations of the Commission on Post-War Plans furnish a "point of departure" from which to begin

Most Frequently Reported Practices in Providing for Individual Differences within the Classroom*

Practice	Number	Per Cent
Differentiated assignments	67	86
Special lessons for weaker students	54	69
Special lessons for stronger students	33	42
Special reports on current topics	14	18
Small groups within the class	29	3
Working on different aspects of same topic	17	22
Working on different assignments	12	15
Projects	34	44
Construction of visual aids	28	36
Demonstration of mathematical principles	7	9
Scrapbooks on uses of mathematics	12	15
Notebooks (principles, examples, sketches)	22	28
Taking charge of bulletin boards	9	12
Working almost entirely individualized	8	10
Supervised study within the classroom	42	54
Use of Strathmore Sheets (Problem-solving group)	11	14
Use of Arith-o-Cards and other games	7	9
Use of much supplementary work	62	80
Duplicated materials	42	54
Different sets of books; "reading corner"	15	19
Use of many diagnostic tests	49	63
Use of resource units	14	18
Sections working with "adapted materials"	3	4

* This table includes replies from seventy-eight classroom teachers of non-traditional mathematics courses.

The first line of the table should be read thus: Sixty-seven teachers, comprising eighty-six per cent of those giving information regarding classroom practices, were using differentiated assignments as a means of providing for individual differences within the classroom.

a thorough reorganization of the mathematics program. Two of the specialists also recommended that the mathematics courses be "ungraded" at the high school level.

Eighty-one of the cooperating schools reported increasing mathematics enrollments, and eighty-six reported decreasing student failures in mathematics classes. Eighty-eight of the schools (96 per cent) reported definite guidance programs and well defined procedures for selecting and enrolling students in the various mathematics courses. The reported plans varied from simple, commonly used methods to rather complex procedures, and most of them included a consideration of the following items: (a) the student's past achievements, (b) teachers' recommendations, (c) reading comprehension test results, (d) special interests and vocational preferences, (e) I.Q. scores, and (f) parents' wishes. The cooperating specialists agreed that adequate guidance is obviously a necessity for the success of any educational program and that informed counseling will provide an important means toward attaining the objectives of non-traditional mathematics courses.

Regarding in-service teacher education programs, the coöperating specialists agreed that a study of the following topics would prove valuable aids to better teaching: (a) the nature of child growth, (b) techniques of guidance, (c) the nature of the learning process, (d) curriculum research, and (e) the nature of the community and of the vocational opportunities within the community.

Respondents from seventy-two of the

cooperating schools reported "administrative encouragement" of curriculum revision in their schools; however, only thirty-six indicated that the time for such work was provided by the school. In thirty-seven schools, space was provided for such work; in thirty-two schools a professional library and materials were provided; and in twenty-four schools secretarial services were provided. In seven of the schools, three of the above-mentioned provisions were made; and in five instances, all four provisions were made by the school.

Other findings of the investigation dealt with the use of workshops, the building of resource units, class size, the qualifications for teachers of the non-traditional mathematics courses, provisions for individual differences within the classroom, the use of teaching or learning sequences in arithmetic classes, and methods of evaluation.[2] It may be noted here that the majority of the questionnaire respondents, as well as of the cooperating specialists, agreed that the area of evaluation offers a wide field for further research.

Recommendations

1. There is need for an Articulation Committee—on a state-wide level, if possible—to consider means of developing a sequential and closely coordinated program of mathematics instruction extending from the kindergarten through high school or junior college.

2. There should be a Mathematics Program Committee at the local level, including—if possible—a member of the Articulation Committee, representatives from the elementary schools, all interested members of the mathematics department of the junior and senior high schools, representatives from other departments of the school—especially from the industrial arts and business education departments, adults from the community, and two or three students, with provisions for later inclusion of other school personnel who become interested as the work progresses.

3. The school should provide, within whatever limits are financially possible, school time, a professional library and materials workshop, secretarial services, and the advice of competent consultants for the work of curriculum revision.

4. Curriculum revision, the building of resource units, and the development of teaching sequences should be a part of the in-service teacher education program.

5. Secondary schools should offer a multiple-track mathematics program; sequential, ungraded courses in non-traditional mathematics should be organized; qualified mathematics teachers should be used for instituting core courses or modified core sections; and the traditional mathematics sequence should be improved, with the label of "college preparatory" being dropped from such courses.

6. In connection with both curriculum and course revision, administrators and department chairmen should recognize the necessity of a continuous cycle of trial, evaluation, and reorganization.

7. Teachers and counselors should become aware of the educational philosophy underlying the teaching of non-traditional mathematics courses in the secondary school, and they should understand the major objectives of each course, or sequence of courses, in the mathematics program.

8. Varied criteria should be used in the selection of subject matter content for the courses in each "track":
   (a) Current, successful practices
   (b) The nature of child growth and development
   (c) The recommendations of specialists
   (d) The demands of society, both at present and in the near future

9. Varied criteria should be used in the organization of subject matter content and

[2] Teachers and department heads wishing further information regarding the other findings of the reported investigation may send a stamped, self-addressed envelope and their inquiry to Dr. Lee Irvin, P.O. Box 102, Rosemead, California.

activities for the courses in each "track":

(a) Mathematics should be taught as a system of related ideas (logical organization)

(b) Consideration must be given to the nature of the learning process (psychological organization)

(c) Applications should be chosen which are within range of the students' social and economic understandings (sociological organization)

10. The work offered in each course should be sufficiently varied to meet individual student needs. Mathematical experiences should be provided which will serve (a) to develop skills and understandings in the fundamental processes, (b) to contribute to the common learnings program, (c) to further the growth of special interests, and (d) to provide educational and vocational guidance.

11. Mathematics teachers should give more attention to developing basic number concepts and principles, number relationships and relationships of the fundamental processes, means and uses of approximation, and methods of problem-solving. They should understand the use of a spiral treatment of subject matter, and they should give more attention to the matter of adapting teaching procedures and methods of evaluation to the types of students taught.

12. Mathematics teachers should realize that evaluation is a continuous process involving numerous personal conferences and observations of attitudes and behavior as well as including paper-and-pencil tests. Evaluation must be concerned with individual growth resulting from personalized instruction rather than with achievement comparisons between members of a group. More effort should be put into the development of tests and other means of measuring growth in terms of the objectives of modern mathematics education.

Future Supply of Science and Mathematics Students

(Continued from page 229)

committee which wrote the fellowship recommendations of Dr. Bush's *Science, the Endless Frontier*. This is their statement:

> The uses to which high ability in youth can be put are various and, to a large extent, are determined by social pressures and rewards. When aided by selective devices for picking out scientifically talented youth, it is clear that large sums of money for scholarships and fellowships and monetary and other rewards in disproportionate amounts might draw into science too large a percentage of the Nation's high ability, with a result highly detrimental to the Nation and to science. Plans for the discovery and development of scientific talent must be related to the other needs of society for high ability; science, in the words of the man in the streets, must not, and must not try to, hog it all. This is our deep conviction, and therefore the plans that we shall propose herein will endeavor to relate the need of the Nation for science to the needs of the Nation for high-grade trained minds in other fields. There is never enough ability at high levels to satisfy all the needs of the Nation; we would not seek to draw into science any more of it than science's proportionate share.[4]

As a matter of psychological science it would be interesting to be able to differentiate those high school students who have the potential for successful careers in science and mathematics from those who have the potential for successful careers in other fields. But as a matter of social policy it would be preferable to identify those high school students who have the potential for successful careers in any of the fields of advanced training and to provide opportunities for their development, letting each be influenced by his own wishes, by the competing attractions of a variety of fields, and by his own decisions as to which career is best for him. Within such a program there would be many who would enter the sciences. Science would therefore profit as much from the broader program as from a narrower one, but the country as a whole would profit more.

[4] V. Bush, *Science, the Endless Frontier.* (Washington, D. C.: Gov't Printing Office, 1945), page 136.

General Education Values of Mathematics and the Attempt of a Faculty to Teach Them

By John R. Abernethy
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Mathematics in general education rests upon these axioms. We believe: (1) that mathematics contains values that everyone needs; (2) that most people are capable of acquiring needed values from mathematics; and (3) that the worth of the individual justifies the effort required to give him or her these values.

As our *first general education value of mathematics*, we consider the basic factual information and skills that make mathematics a tool for everybody, that make it a key to other knowledge, and that make mathematics the queen of the sciences because it has become the servant of all. Everyone needs some mathematics as a tool and, for this reason, it deserves a place in the general education program. We take pride in the contribution that mathematics has made to our civilization, in the comforts it has brought to mankind, and in the increased production it has given to agriculture and industry. We see in mathematics a tool by which additional comforts and production will be brought to us in the future. We also see it as a key that will unlock personal opportunity for many in a widening range of human achievements. But even in non-mathematical areas of service there is a certain body of information and skill that must be a part of every well educated person, whatever his specialty.

The tool needs of an individual in mathematics may be compared with the tool needs of an individual in farming. Just as a spade will be satisfactory in digging a garden, but a plow will do a better job, so a certain amount of arithmetic will do for daily living, but more mathematics is desirable when it comes to balancing a bank statement and filling out a government form. Regardless of the mathematical attainment of the student, a greater proficiency in mathematics is an open door to greater opportunity.

As our *second general education value of mathematics*, we consider the aid it can give in personal adjustment. Too often mathematics has been a hindrance in this respect rather than a help. Frequently there is a psychological block. For example, in the teaching of agricultural mathematics, we find students who failed algebra in high school. As long as they are being taught the applications of arithmetic, they progress satisfactorily, but when we start teaching the applications of algebra, we find that they "block." They never have learned algebra, they never expect to learn algebra, and they freeze mentally in the presence of anything resembling algebra. When additional training in mathematics was made available at the high school level, it was soon discovered that not all students were equal in either interest or ability. Classes became unbalanced as many children could not or would not absorb the traditional education of the scholastics. Today we are interested in the good that the individual gets out of his education. We are also interested in the value an educated man can be to his community. We want well integrated persons of self-confidence and poise. Too often the individual who has been endowed with little natural aptitude for mathematics has been exposed to still less inspired and interested guidence in its bewildering intricacies. This student gradually congeals into an attitude of "I am dumb and nobody but a really smart person can make any sense out of mathematics, no matter how hard he tries." Just as a series of patient and efficient boxing lessons will transform the personality of the timid

little boy in the tough neighborhood, so the patient and efficient instruction in some of the fundamentals that can be mastered will transform the personality of the mathematically timid. If mathematics is to fill its place adequately in the general education program, it must help these students to better personal adjustments by giving them tasks which they can accomplish successfully and on which they can grow.

As our *third general education value of mathematics*, we consider the experience it offers in unprejudiced and logical thinking. Mathematics offers the only field of endeavor that operates without prejudice, in favoring neither race, creed, economic position, or form of government. It can be practiced without any emotional disturbance, and international understanding is at its greatest in the field of mathematics. Two plus two equals four even when it is translated into Russian. Critical thinking, values, and judgment form a major emphasis of general education. The colleges are now seeking to train young people who are potential homemakers and contributors to the religious, social, and political activities of their communities. If this training is to be effective, it must contain attitudes of respect for the opinions of others in developing value judgments, and fairness in judging controversial matters taking into consideration all sides of the subject. Fair mindedness is a definite aspect of mathematics, and in our program of general education, we should find mathematics upholding truth, integrity, and honesty, for they are at the heart of the subject.

As our *fourth general education value of mathematics*, we consider its esthetic value. A somewhat extravagant expression of this value is given in Edna St. Vincent Millay's poem, "Euclid Alone Has Looked on Beauty Bare."[1] The esthetic value of mathematics is more than its tool value in the fine arts. It is the sense of value that comes from looking at form and symmetry, that comes from listening to harmony of sound, the sense of satisfaction that comes from observing the harmony of mathematical relationships, in the appreciation of elegance in the work of colleagues, in working out intricate problems successfully, and in the utilization of creative ability in mathematical research. New mathematics is usually created because of an overwhelming interest in the subject itself rather than in answer to some specific problem in science or engineering. It was only after the tensor calculus had been known for a quarter of a century that it was used in the development of the general theory of relativity.

But we should be speaking here about the esthetic value of mathematics in general education. Besides the particular professional and vocational groups, mathematics has some esthetic value for everyone. From *Mathematics in Aristotle*, we read: "For the chiefest forms of the beautiful are orderly arrangement, symmetry, and definiteness and the mathematical sciences have these characters in the highest degree. And since these characters, such as orderly arrangement and definiteness, are the causes of many things, it is clear that mathematicians could claim that this sort of cause is in a sense like the beautiful acting as a cause." Every student can find something of interest and something to appreciate among the number of books now available which present some of the finer things of mathematics in quite readable form. The sense of satisfaction of a job well done, a problem completed to perfection, or the gratification of intellectual curiosity is of course independent of its classification by college level.

Our *fifth general education value of mathematics* is the insight it gives us into the knowledge and understanding of our cultural heritage. We may consider this fifth value under the title "Mathematics

[1] Edna St. Vincent Millay, "Euclid Alone Has Looked on Beauty Bare," *Chief Modern Poets of England and America*, ed. G. Sanders and J. Nelson (New York: Macmillan Co., 1929), p. 626.

as a Culture Clue" (c.f. the book of this title by Cassius Jackson Keyser), but more than that, mathematics is a primary driving force in the development of culture. It is only recently that mathematics, science, philosophy, and religion have been separated. We go back to philosophical journals for the first printed papers on mathematics. Newton's great work on mathematics and physics contains in its title the term "philosophy." Pythagoras, in his lectures, taught mathematics together with its application to music, astronomy, and philosophy. His numerology was the means by which he claimed to prove the existence of God from observation by means of reason. The fault was not with the aim of Pythagoras, but with his meager knowledge of mathematics.

As with religion, the beginning of mathematics is God. Back of the creation is the design, and the design is mathematical. When the ancients discovered mathematics or science, they correctly attributed it to God; and their knowledge became a part of their philosophy. Mathematics was taught alongside the principles of morality. Today we will class as superstitution much that was taught under the name of mathematics, but let us take care lest we throw out too much. Mathematics today supports the existence of God and morality on a much sounder foundation than the mystery societies of ancient Egypt or the numerology of Pythagoras. William T. Meyer in the report of the Fifth Annual Conference on Higher Education deplores the lack of attention paid to religion by higher education in America.[2] Mathematics cannot take over the task of teaching religion, but it can, without fear of contradiction, deal with those phases of religion that come under its jurisdiction. It can demonstrate the absurdity of the assumption that the world exists as we know it by chance alone. Even the idea of immortality is essentially a mathematical concept. Beginning with the primitive who first thought of the needs of tomorrow, the mathematical concepts of order helped to develop an agricultural economy and thinking that looked ahead to a future harvest and a still more future need. In extending this concept, man next thought of better preservation of his usefulness by an unbroken continuity of sons and the sons of sons, and finally of preserving his immortal soul in the Infinite Kingdom of God. And where but in the field of mathematics are we adequately prepared to deal with the Infinite?

We have gone through the objectives of general education, and we find that for every objective there is a value in mathematics that everyone needs. Our second axiom is that nearly everyone is capable of receiving these values. American democracy is attempting to bring the benefits of an education to a much larger and a more varied body of students than has ever been reached in the past. At the Fifth Annual National Conference on Higher Education the Commission findings were as follows: "There are not too many students attending college today. On the contrary, the colleges of the nation are not producing a sufficient number of well educated persons to meet the needs of a democratic society."[3] "Findings of the the Commission indicate that 49% of American youth have intellectual abilities sufficient to enable them to profit by at least two years of college training and that 32% have sufficient ability to warrant their attendance for four or more years."[4] But the question before us is not so much what percentage of our youth can take mathematics

[2] William T. Meyer, "Religion in Higher Education," *Current Issues in Higher Education, 1950* (Report of the 5th Annual National Conference on Higher Education [Washington, D.C.: N.E.A., Department of Higher Education, 1951]). p. 113–18.

[3] A. L. Vaughan, "Who Should Go To College?" Section B. (Report of the 5th Annual National Conference on Higher Education, p. 37). See footnote 2.

[4] Charles E. Atkinson, "Who Should Go to College?" Section A. (Report of the 5th Annual National Conference on Higher Education, p. 32). See footnote 2.

as what percentage of those who can make a go of other courses in school can reasonably profit by taking courses in mathematics also. In partial answer to this question, I wish to present what we have tried to do at Arkansas Polytechnic College and what results we have had. I wish to express my appreciation to a cooperative administration in the college for making this experiment possible and providing us with proper equipment for the job.

One group of our students enters college with a high comparative proficiency in mathematics. In this group are engineering students who must complete the calculus within two years as part of the nineteen semester hours requirement in mathematics. Others in the group are mathematics majors who plan either to teach in the high schools of our state or to enter a graduate school. Those capable of doing this work deserve the chance of learning in competition with their peers in courses that have not been watered down to suit the pace of the slower students. To these we offer the standard mathematics courses stepped up in content and value through more adequate placement of topics.

A second group of our students can learn the same material that the first group covers if given more time in class. For these we offer a course in intermediate and college algebra with five hours in class for three hours credit. Our experience indicates that there is no absolute minimum proficiency prerequisite for this course. With determination and hard work, there will be some students who are able to make good on this course although they are not reasonably prepared for it. The general requirement is a minimum proficiency equivalent to a year of high school algebra and one of plane geometry.

Our third group of students enter college with a very low proficiency in mathematics. They are not ready for any algebra course that could possibly fit a person for calculus. It is in this third group that we find the greatest need of the general education values of mathematics, as well as the greatest test of our second axiom. For these we offer a new course in "Basic Mathematics." Students in this course meet five hours per week but receive only three hours credit. Starting with inventory tests on basic arithmetic, drill exercises are assigned as needed and both tests and drills are completed without error before the student advances. Each student progresses at his or her individual pace. Just like in an electrical circuit, their mathematical learning needs a certain continuity. If there is a break in their power line, no amount of patching up somewhere else will be effective; it is necessary to go back to the break, repair that, and then proceed from there. Too often expert teaching of a new topic in mathematics is wasted because the student does not have the foundation upon which the new is to be built. If we can go back, find these breaks in training and repair them, we then have a student ready to go on to new topics.

The results have been gratifying. By the end of the semester, students with an average proficiency[5] of 4.8 advanced to an average of 11.1. This gain of 6.3 points in one semester was more than their entire former training. Two-thirds of the students entering this course unprepared for intermediate and college algebra, moved up to that level with average proficiency increasing from 5.4 to 13.5 points. One third of the remaining group showed marked improvement in proficiency with increases of from 4 to 9 points to reach scores of 7, 8, and 9. This is especially gratifying when we consider that the standard expectation of these students in mathematics is an F or a low D.

We realize that our experience is young and that the amount of coverage is small. At the same time we have seen the student who has been dormant mathe-

[5] Measured by the score on the Cooperative Mathematics Pre-test for College Students, Form X. Princeton, N. J.: Educational Testing Service.

matically now blossoming before our eyes. We believe that most students are capable of acquiring needed information and skills in mathematics and that our results are representative of what can be done in general and not just the fortunate experience of a few teachers with a few students.

Of all the values of mathematics in general education, we are probably doing our best job in the field of personal adjustment. It has been said that the slowest part of the general education program is the development of its testing program. We are able to report objectively with fair accuracy what we are doing in developing skill and imparting information, but our measure of the good we are doing in the field of personal adjustment must remain largely subjective. Yet we speak with confidence since time after time we have seen students work through to a point where there has been some "block," remain stalled for a while, then take off in creeper gear, and finally shift into high. The student whose chronic complaint has been, "I never could do fractions, or algebra," can now be answered, "No? but you are doing it now." His reply is, "Yes, and I like it." Confidence comes with doing. The student learns to work independently and to rely upon himself. The student learns to complete one task, within allowable tolerances, before he goes on to the next. He learns that he can do a job even when it looks extremely difficult. It may take him a long time, but he finishes it not only with increased information and skill but also with a sense of completeness and satisfaction. Complexes vanish and dispositions mellow.

Because students in our basic mathematics classes become quite interested in developing technical proficiency, most of our class time is spent on basic information and skill, and the students desire it to be that way. But we attempt to spend at least one hour each week in discussing the nontechnical aspects of mathematics and sometimes a group has become so interested in some particular question that time has been profitably spent in discussing it. Usually students prefer to work exercises and therefore we seek to give the logical, esthetic, and social values of mathematics primarily by independent outside readings to be evidenced by sixteen reports of about three hundred words each either written or oral. Quality ranges widely. Whatever is done is accomplished with the use of very little class time. By giving a wide range of choice, we attempt to create interest. Our own accomplishments on these values are small but the quality of some of these reports indicates that there is a distinct possibility of increasing the social values of mathematics in general education.

There are many philosophical bases for our third axiom. Most teachers would be willing to give it lip service, but this is not enough. We can impart enough values to a sufficiently large group to justify the inclusion of mathematics in the general education curriculum. But while we can teach mathematics to the low proficiency group, we cannot make it easy for them to learn and certainly we cannot make it easy on ourselves as teachers. For this program we need our best teachers; those who have initial high scholastic attainment and who, having caught the spirit of general education, put the student above the subject matter.

The student must truly be the central figure in his education program, with the teacher adapting methods and content to the student's need and being, as Robert A. Johns expresses it, "cognizant of the influence the subject will have upon the student."[6]

Not only does the individual student need values, facts, and processes found within the bounds of mathematics, but society also requires that he have them. As Harold W. Stoke expresses it (Fifth

[6] Robert A. Johns. "The Student as a Factor in his Own Education." (Report of the 5th Annual National Conference on Higher Education, p. 57). See footnote 2.

(*Continued on page 259*)

A Logical Symbolism for Proof in Elementary Geometry

By WALLACE MANHEIMER
Franklin K. Lane High School, Brooklyn, New York

THE PUPIL who draws dashes through lines or angles in a geometric diagram to denote equality by hypothesis is taking advantage of an elementary logical symbolism. Ever since the days of Boole and Frege such symbolisms have become increasingly important in representing and furthering mathematical thought. Present day symbolic logic, which is an outgrowth of the methods of logical symbolism, has begun to revolutionize such diverse fields as the design of mathematical instruments, insurance analysis, brain physiology, and electrical network design. The study of symbolic logic at present occupies the full time research of at least two hundred creative mathematicians in our universities and industrial laboratories.

There has been widespread popular interest in these modern developments. The remarkable powers of mathematical computing machines have struck the imaginations of many of our pupils. A good deal of the growing literature of "science fiction" has been devoted to related subjects. Popular magazines, such as the *Scientific American* have featured articles on symbolic logic and its applications.

In order to reveal to the high school student some of the potentialities of logical symbolism, a simple application of it was attempted in a geometry class. Since it proved to have many values in the instruction of geometrical proofs, a brief summary of its methods will be given here. The symbols were a set of labels, each of which represented both a statement and reason. For example, the familiar dashes were retained for equality by hypothesis, and any other symbols that were found to be in current use were also incorporated into the system. The completed proof consisted only of a labeled diagram, in addition to the listing of hypothesis and conclusion. In order to keep in contact with other classes, oral presentations of the proofs were made in the traditional manner.

A number of interesting and rather unexpected advantages of the new method soon came to light. However, the reader first should study some of the actual symbols that were used, together with several proofs that will feature them. They are listed on the following pages.

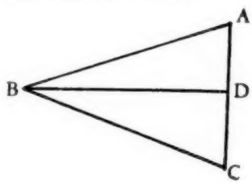
There were other symbols and, like those above, they were made as nearly self explanatory as possible.

A few proofs of increasing complexity will now be given. To save space the usual solutions will not be included, but the reader can, of course, make his own comparisons with the traditional form.

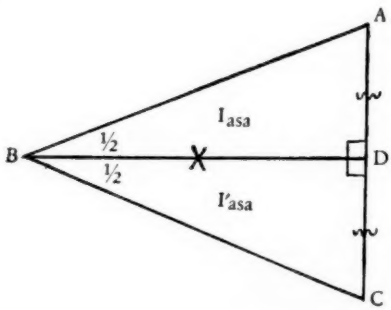
Problem 1.

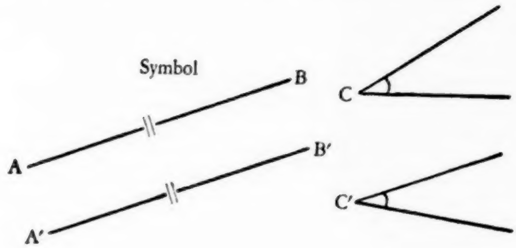
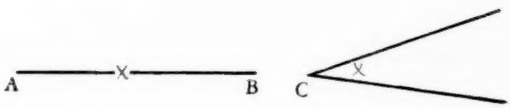
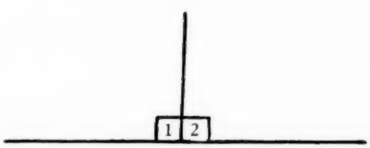
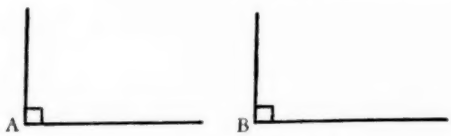
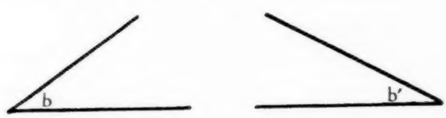
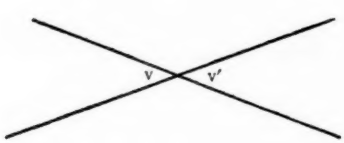
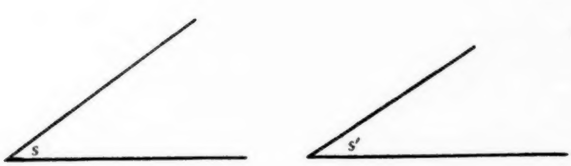
Hypothesis: BD bisects ∠ABC
BD is perpendicular to AC

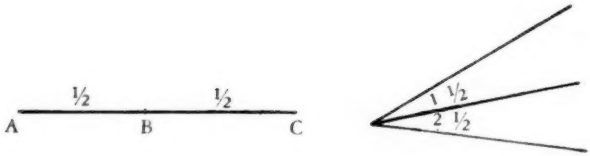
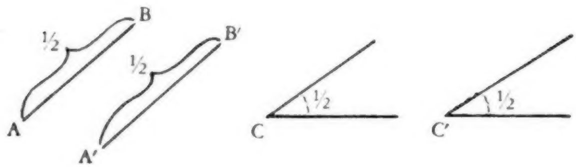
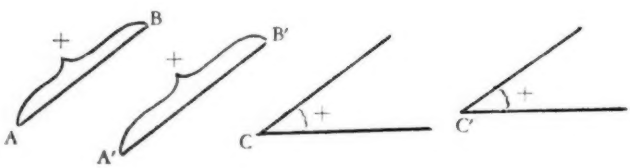
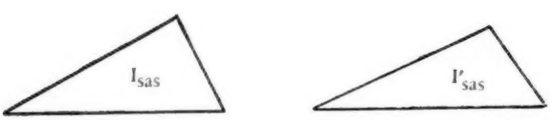
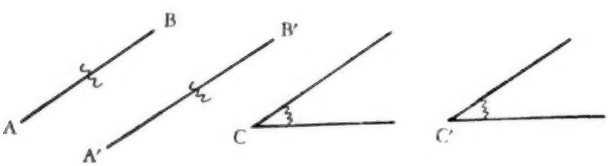
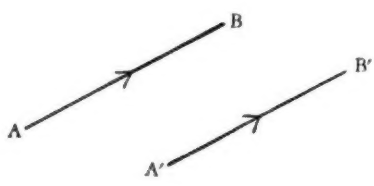
Conclusion: AD = DC

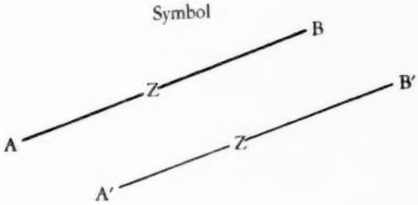
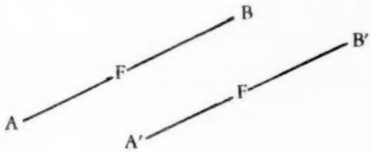
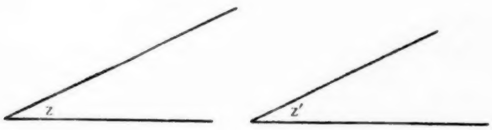
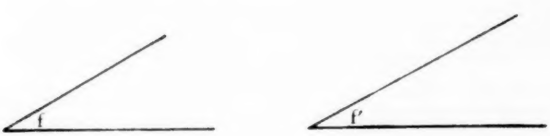


Proof;



Symbol		Translation
	$AB = A'B'$ $\angle C = \angle C'$	Hypothesis
	$AB = AB$ $\angle C = \angle C$	Identity
	$\angle 1 = \angle 2$	Perpendiculars form right angles; all right angles are equal.
	$\angle A = \angle B$	All right angles are equal.
	$\angle b = \angle b'$	Base angles of an isosceles triangle are equal.
	$\angle v = \angle v'$	Vertical angles are equal.
	$\angle s = \angle s'$	Supplements of equal angles are equal.

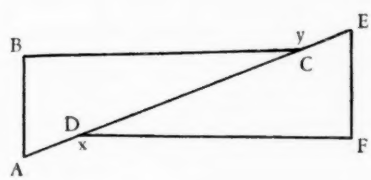
Symbol	Translation	
	AB = BC $\angle 1 = \angle 2$	Bisection
	AB = A′B′ $\angle C = \angle C'$	Halves of equals are equal.
	AB = A′B′ $\angle C = \angle C'$	If equals are added to equals the results are equal.
	$\triangle I \cong \triangle I'$	Side, angle, side, etc.
	AB = A′B′ $\angle C = \angle C'$	Corresponding parts of congruent triangles are equal.
	AB ∥ A′B′	Hypothesis

Symbol		Translation
	$AB \parallel A'B'$	Two lines are Parallel if a pair of alternate interior angles are equal.
	$AB \parallel A'B'$	Two lines are parallel if a pair of corresponding angles are equal.
	$\angle z = \angle z'$	If two lines are parallel alternate interior angles are equal.
	$\angle f = \angle f'$	If two lines are parallel, corresponding angles are equal.

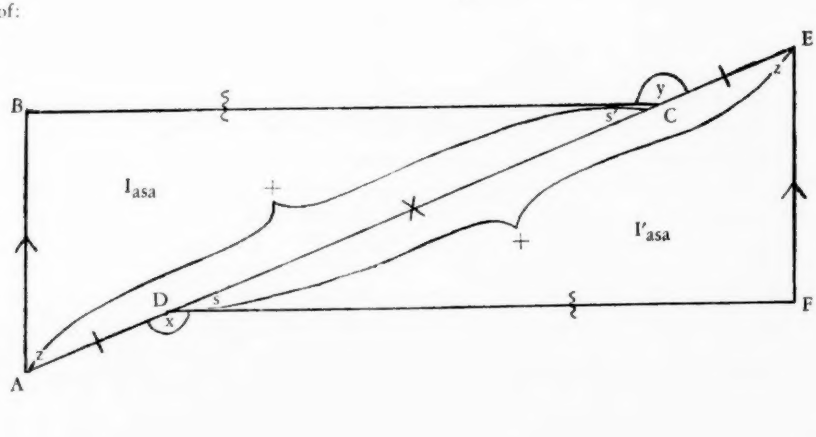
Problem 2.

Hypothesis: $AD = CE$
$\angle x = \angle y$
$BA \parallel EF$

Conclusion: $BC = DF$



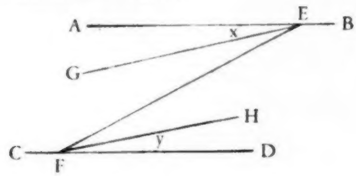
Proof:



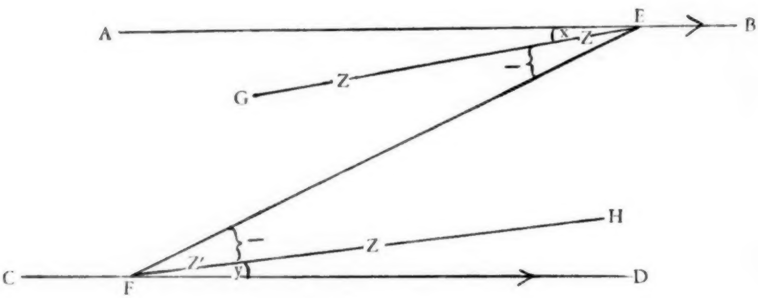
Problem 3.

Hypothesis: $AB \parallel CD$
$\angle x = \angle y$

Conclusion: $EG \parallel FH$



Proof:



It will be observed that the sequence of steps must be supplied by the reader. This was found to have advantages as well as disadvantages in classroom work. In more difficult proofs, especially those involving more than one pair of triangles, the basic order of steps was revealed by using sequential diagrams. The symbol for "hypothesis" was used for parts obtained from a preceding diagram and was then read, "see previous reason."

Problem 4. Corresponding medians of congruent triangles are equal

Hypothesis: $\triangle ABC \cong \triangle A'B'C'$
CD and $C'D'$ are medians.
Conclusion: $CD = C'D'$

(*See figure at top of next page*)

The last example is one frequently asked as an "honor" problem early in the first term of geometry. The symbolic notation made it much easier for even pupils of moderate ability to follow.

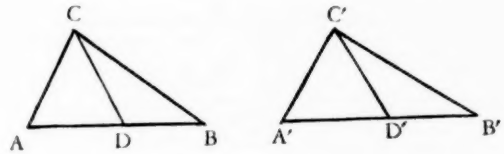
Problem 5

Hypothesis: $AB = AC$
$DB = EC$
Conclusion: $BG = GC$

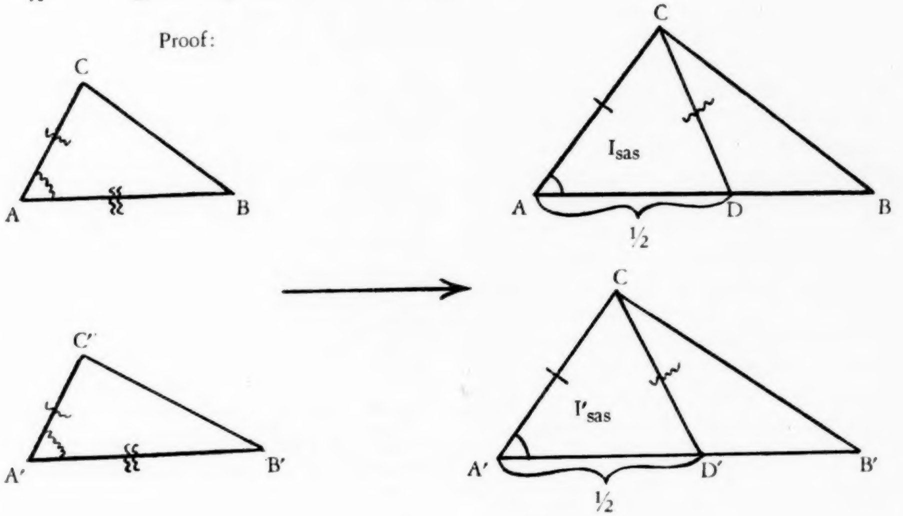
(*See figure at bottom of next page*)

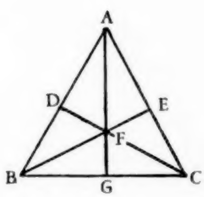
The symbols, as described above, carried the class well into the second third of the term. At this time they were dropped, partly with a view toward preparation for a uniform final examination. The class had, by this time, become very fond of the symbolic method and there was universal regret at the return to the traditional written form.

Even after discounting the pupils' natural interest in a novelty—that imponderable that is so often responsible for the "success" of educational experiments—the symbolic method appeared to have definite pedagogic advantages. It enabled the class to study a much larger number of problems at the blackboard and in homework. This was especially valuable near the start of the term, when exposure to a great variety of problems reinforced the class's understanding of the nature of proof and its application to geometrical data. Pupils also pointed out that they could solve problems more easily by "thinking on the diagram," as one of them described it. The symbolism did appear to simplify some of the complex psychological operations involved in formulating a proof. It enabled pupils to record their

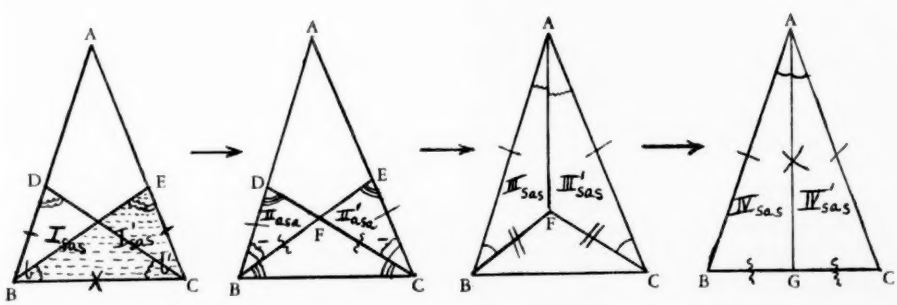

C
A
D
B
C′
A′
D′
B′

Proof:


C
A
B
C′
A′
B′
C
I_{sas}
A
D
B
½
C
I'_{sas}
A′
D′
B′
½


A
D
E
F
B
G
C

Proof:


A
D
E
I_{sas}
I'_{sas}
B
C
A
D
E
II_{asa}
II'_{asa}
F
B
C
A
III_{sas}
III'_{sas}
F
B
C
A
IV_{sas}
IV'_{sas}
B
G
C

reasoning as they proceeded, it concentrated their reasoning on a single diagram, and it did away with the phobia toward the written word that possesses some of our weaker students.

Beyond this, however, there were greater values. The opportunity was open to show the uses of mathematics in a new and thoroughly modern way. Many pupils read John E. Pfeiffer's interesting article on symbolic logic in the *Scientific American,* as well as others on computers, "mechanical brains," chess playing machines, and the application of logical symbolism to a variety of fields. One interested pupil read the introduction to Whitehead and Russell's *Principia Mathematica,* in which there is set forth the symbolic logic that is used in this great work. Thus the experiment succeeded in arousing deep and perhaps lasting interest in mathematics.

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Weaver, Warren. "The Mathematics of Communication." CLXXXI (July 1949) 11–15.

Shannon, Claude. "A Chess-playing Machine." CLXXXII (February 1950) 48–51.

Berkeley, Edmund C. "Simple Simon," CLXXXIII (November 1950) 40–43.

Pfeiffer, John E. "Symbolic Logic," CLXXXIII (December 1950) 22–24.

Walter, W. Grey. "A Machine that Thinks." CLXXXV (August 1951) 60–63.

Gardner, Martin. "Logic Machines," CLXXXVI (March 1952) 68–73.

"Automatic Control" issue, CLXXXVII (September 1952). Contains a wealth of articles on modern developments of symbolic logic.

Workshops and Institutes

The Committee on Family Financial Security Education has announced that three hundred and thirty scholarships will be available for the summer of 1953 for high school teachers and educators who are interested in developing better ways of teaching family financial security in American classrooms. The universities where the workshops will be held comprise Connecticut, Denver, Miami, Oregon, Pennsylvania, Southern Methodist, Virginia and Wisconsin. Information about the workshops and scholarships may be obtained from R. Wilfred Kelsey, secretary of the Committee on Family Financial Security Education, 488 Madison Avenue, New York 22, New York.

The Department of Mathematics at Teachers College, Columbia University announces the following courses for the session for July 6 to August 14: Professor Fehr, teaching arithmetic in the elementary school; Professors Fehr and Rosskopf, research and departmental seminar in teaching mathematics; Professors Fehr and Yates, professionalized subject matter in advanced secondary school mathematics, part I; Professor Rosskopf, foundations of mathematics for teachers; Professors Rosskopf and Shuster, teaching geometry in secondary schools; Professors Rosskopf and Yates, teaching of elementary college mathematics; Miss Schult, supervision and teaching of mathematics in the junior high school; Professor Shuster, field work in mathematics; business arithmetic and mathematics.

A special workshop in the theory, construction and use of models and materials in mathematical education will be held July 20 to 31 under the direction of Professors Fehr, Yates and special lecturers.

The University of Arkansas, with the cooperation of the **Arkansas Council of Teachers of Mathematics,** will conduct a **Workshop in Mathematics,** July 13–15, 1953. Consultants will be Miss Martha Hildebrandt, Proviso Township High School, Maywood, Illinois, past president of the National Council of Teachers of Mathematics; Professor Maurice L. Hartung, Department of Education, The University of Chicago; Miss Christine Poindexter, Senior High School, Little Rock, Arkansas; Mrs. Virginia Sue Wilson, Supervisor of Elementary Education, College of Education, University of Arkansas. There will be general meetings and discussion groups for secondary teachers and for elementary teachers.

The University of Arkansas, at Fayetteville, is in the Ozark Playground area, at an altitude of 1450 feet above sea level. There are many accommodations for tourists throughout the area. Participants in the Workshop may obtain ac-

(*Continued on page 296*)

The Englishman's Mathematics as Seen in General Periodicals in the Eighteenth Century*

By Norman A. Goldsmith
Henderson State Teachers College, Arkadelphia, Arkansas

I

To the historian of mathematics, the eighteenth century extends from Newton through Euler to Gauss. Newton's *Principia* appeared in 1687. Only four years later England had its first periodical of general interest and long life, *The Athenian Mercury*. Thus, in their early years, the new mathematics and the new magazines grew together in a restless atmosphere as science, commerce, and political reform exhibited a vitality never known before.

The history of mathematics in the schools and laboratories of that century has been well told and need not be repeated. Rather, the concern of this paper is with the mathematics of the educated eighteenth century Englishman, not trained in mathematics, who had enough learning and leisure to read the newly developing periodicals.

The source of materials is the periodicals themselves, embracing a period from 1703 to 1816. Reference is made to them with four purposes: first, to describe the mathematics of the non-scientific magazines; second, to report some facts relative to the mathematical learning of the eighteenth century Englishman; third, to point out that mathematical subjects had a large body of readers; and, fourth, to show that the effort to keep mathematics before the public had strong support from the magazines.

II

Relative to the first of these purposes, consider the title of a monthly founded in July, 1747: "*The Universal Magazine of Knowledge and Pleasure*, Containing News, Letters, Debates, Poetry, Musick, Biography, Voyages, Criticism, Translation, Philosophy, Mathematicks, Husbandry, Gardening, Cookery, Chemistry, Mechanicks, Trade, Navigation, Architecture, and other Arts and Sciences Which may render it Instructive and Entertaining to Gentry, Merchants, Farmers, and Tradesmen. To Which will occasionally be added An Impartial Account of Books in several Languages and of the State of Learning in Europe; Also of the stage, new Opera's, Plays, and Oratorio's."

In this catalog of departments, "Mathematicks" has eleventh position. However, to many eighteenth century readers, as well as to their twentieth century counterparts, "Mathematics" meant number puzzles. One such puzzle was submitted to the *New Monthly Magazine* in these words: "To find out four numbers that may be equal to a number taken, and such, that the difference between any two of them, may be a square number."[1] The proposer repeated almost verbatim the language in which the same problem had been submitted to the *Universal* sixty-six years before.[2]

One more problem is worth quoting for the flavor of the language: "How much does a man's Head walk more than his Feet, supposing him to travel a thousand miles?"[3]

Not all problems had the puzzle flavor.

* Presented at the Thirteenth Christmas Meeting of the NCTM, December 30, 1952, at Lincoln, Nebraska.

[1] *New Monthly Magazine* (1814), II (June 1814), 398. (The number in parentheses after the name of the journal is the date of founding, according to Graham. See footnote 37.)

[2] *Universal Magazine of Knowledge and Pleasure* (1747), II (April 1748), 175–77.

[3] *Athenian Oracle* (1701), III (1704), 470.

In fact, problems dealing with maxima and minima were very popular. One which asked the dimensions of the (right circular) cylinder of diagonal forty inches and maximum volume was, fittingly enough, solved by a cooper.[4]

Problems in gravitational forces were equally popular, like this one: "Suppose the diameter of the earth to be 8000 miles and a hole perforated thro' it. In what time will a ball projected from the center arrive at the superficies, and with what velocity must it be projected?"[5] A solution to this problem was submitted by a schoolmaster, and, four months later, a complaint from the proposer that it was "absurd" accompanied the correct solution.

Geometric series fascinated the readers. One of the earliest references is this: "Suppose a Bullet should fall down eternally and nothing should ever interpose; and the manner of its falling should be thus; the first Minute it should fall 20 Miles, the second Minute 19 Miles, the third Minute $18_{10}{}^{3}$ (sic) Miles, and so onward forever in the same Geometrical Progression; I demand how far it will fall in a whole Eternity." The editor answered, ". . . 400 miles, which is the Solution of the Question, however strange and surprising it may seem to some Persons, who are not acquainted with Mathematical Demonstrations."[6]

Number theory also attracted intelligent questions and answers. The *Athenian Oracle* for 1704 was asked what are perfect numbers and replied with the equivalent of Elucid's formula.[7] This was sixty-six years before Euler proved that all even perfect numbers are obtained from this formula.

If these questions have a familiar sound to mathematics teachers today, so do many others. Here are some from the *Athenian Oracle* of 1703:

[4] *Universal Magazine of Knowledge and Pleasure*, XII (April 1753), 167–70.

[5] *Ibid.*, X (January 1752), 29.

[6] *Athenian Oracle*, III (1704), 474.

[7] *Ibid.*, p. 248–49.

"Whether One be any Number?"[8] "Whether Quadrature of the Circle be Possible?"[9] Why are all number systems based on 10?[10] "Is infinite number a contradiction?"[11] "Are infinite numbers equal?"[12]

Of the famous eighteenth century mathematicians, Thomas Simpson was the most regular contributor to the popular journals. He and four other prominent mathematicians were asked by the *Universal* to comment on the relative strength of elliptic and semi-circular arches for the new "Black-Fryars" bridge. Only Simpson took much care with the answer, and, in his letter of transmissal, he said, "When opportunity shall permit, I purpose to lay the whole of my thoughts and demonstrations on this subject before the Royal Society."[13]

Simpson made frequent use of general magazines to bring mathematics before the public. From 1754 to 1760 he served as editor of *The Ladies' Diary* (founded in 1704), a connection terminated by his death. Twelve or fifteen problems, some of them very difficult and near the existing frontier of mathematics, appeared in each issue.

In November, 1736, Simpson contributed to the *Gentleman's Magazine* a hitherto unpublished solution of one of Leibnitz's problems on infinite series. So great was his prestige that he was permitted to defend his paper against an attack signed "Mr. Facio." Simpson in turn proposed another problem and in doing so extended a series of mathematical articles that continued in this monthly through twenty-six consecutive issues.

In the course of this time,[14] he presented a question on finding the ratio of the equa-

[8] *Ibid.*, I (1703), 355.

[9] *Ibid.*, p. 345.

[10] *Ibid.*

[11] *Ibid.*, II (1703), 86.

[12] *Ibid.*

[13] *Universal Magazine of Knowledge and Pleasure*, XXVI (May 1760), 251–54.

[14] *Gentleman's Magazine* (1731), VIII (August 1738).

torial and polar diameters of the earth, admitting that part of his solution was from the Memoirs of the Royal Academy at Paris. He explained that he presented it through the *Gentleman's* for the sake of wider circulation.

Though their papers were occasionally reprinted from the scientific journals, and though their names were mentioned[15] as if they should be familiar, most of the other great mathematicians failed to share Simpson's attitude toward use of the popular magazines. As a result, papers having to do with pure mathematics are rare. In its fifty-three years, the *Universal* printed only ten. One of these was Simpson's preliminary note on the "Black-Fryars" bridge. Another described the construction and use of mathematical instruments.[16]

Among the others, "The Calculations of all the Theorems for solving all questions in Arithmetical Progressions"[17] lists thirty-three formulas based on $S=\frac{1}{2}n(a+l)$. A college freshman could do as well.

"The Calculation of the Credibility of Human Testimonies"[18] uses the theory of probabilities with some arbitrary assumptions to "prove" that one shouldn't believe much of what he hears.

"Ninety Theorems on Properties of the Circle"[19] states without proof theorems now taught in high school classes.

"An Investigation of the Logarithmic Series from the Principles of Fluxions"[20] states and proves the theorem "The fluxion of the hyperbolic logarithm of any number is equal to the fluxion of that number (whose logarithm it is) divided by the number itself." Except for its quaint terminology, it is accurate, concise, and above the usual level.

The *Monthly Magazine* carried a brief paper tracing the history of our digits back to the Greeks of Alexandria[21] and a problem on infinite series.[22]

Meanwhile, Simpson's quondam adversary, Facio, had prepared a paper titled "The Quantity of Refraction of Light in the Moon's Atmosphere determined" in which he declares that calculations of longitude may be as much as four degrees in error due to this refraction.[23]

III

From what has been said it is possible to infer something of the mixed and uneven quality of the mathematical papers presented in the popular eighteenth century magazines. In this formative period for their journals, many editors were willing to accept almost anything that came to hand early enough. Sometimes this policy led to heated disputes between correspondents.

One such dispute centered around Thomas Taylor, a respected scholar. Taylor edited a book entitled *Elements of the True Arithmetic of Infinities*, announcing in the preface that he had "demonstrated all the propositions of Dr. Wallis's Arithmetic of Infinities and also all the principles of the Doctrine of Fluxions, to be false." He referred to Newton as a man of "a rambling and precipitate genius, but a perpetual blunderer."[24]

Taylor's book touched off a long dispute with a good mathematician, W. Saint. Saint expressed his "most profound respect for (Taylor's) abilities but with the deepest regret that these abilities should have been used to the detriment of the mathematical sciences."[25] Taylor, wishing Saint "the possession of a *sound* mind as soon as possible,"[26] resumed his study.

[15] *Universal Magazine of Knowledge and Pleasure*, XIII (December 1753), 274–75.
[16] *Ibid.*, X (May 1752), 228.
[17] *Ibid.*, IV (February 1749), 78–79.
[18] *Ibid.*, (March 1749), 107.
[19] *Ibid.*, X (June 1752), 271–73.
[20] *Ibid.*, XVII (October 1755), 172.

[21] *Monthly Magazine* (1796), XXXII (January 1812), 507.
[22] *New Monthly Magazine*, VI (August 1816), 23.
[23] *Gentleman's Magazine*, VIII (March 1738), 130.
[24] *Monthly Magazine*, XXXI (May 1811), 314–19.
[25] *Ibid.*, XXXII (September 1811), 120–25.
[26] *Ibid.*, (October 1811), 215.

Three years later[27] he had "discovered" the following reduction of the fraction $1/(2+1-1)$:

$$\begin{array}{r|l} & \underline{1/2-1/4+3/8-5/16+11/32-21/64\cdots} \\ 2+1-1) & 1 \\ & \underline{1+1/2-1/2} \\ & \underline{-1/2+1/2} \\ & \underline{-1/2-1/4+1/4} \\ & \qquad\qquad \text{etc.} \end{array}$$

The quotient he grouped in the manner of $(1/2-1/4)+(3/8-5/16)+(11/32-21/64)\cdots=1/4+1/16+1/64\cdots$ and obtained the sum 1/3, not the expected 1/2.

His error was pointed out[28] by a capable mathematician using the pseudonymn "Philomath." Taylor, still unconvinced, replied in the next issue, "Though I have no doubt his reply to me is exactly such as would have been made by the greatest of modern mathematicians, it is certainly by no means satisfactory."[29]

In support of his position, Taylor quoted a misunderstood passage from the blind mathematician Saunderson and the *Philosophical Principles of Religion* by the Reverend Doctor Cheyne. Thereupon Philomath declared, "I shall decline entering the lists with Mr. T., for what mathematician would engage in argument with a person who maintains that $1-1$, $1/2-1/2$, etc., are infinitely small quantities; and that nothing divided by a number will produce a quotient of some value (sic)."[30]

By now the dispute had settled around whether $(0\times0)/0$ is or is not 1. An onlooker, Philotailor, following the custom of the time in reading "nothing" for "0," entered the dispute[31] with the argument that there are two nothings, absolute and relative, and that, since $(1-1)$ is a relative nothing, it may not perform like the number "0," which is unattainable, absolute nothing.

When the argument closed,[32] it had dragged on through ten consecutive issues. Less than a year later,[33] the same magazine announced publication of Taylor's *Theoretic Arithmetic*. No defenders of mathematics rose to challenge it. The magazine continued to print Taylor's articles on half a dozen subjects, culminating in *Evidence of the Truth of Alchymy*.[34] This was as scholarly as the title suggests, and on a par with his mathematics.

The Taylor-Philomath dispute is significant for presenting several characteristic qualities of the mathematical writing of the time. First is the fact that one could establish a reputation as a scholar on the basis of prolixity rather than quality of work. Second, the best mathematicians were reluctant to sign their names to contributions to non-scientific journals. Third, ecclesiastical authority was likely to be accepted as final unless opposed by other similar authority. Fourth, the distinction between "zero" and "nothing" was far from clear. Fifth, the appreciation of rigor and of consistency in mathematical systems had not spread far from the universities, so that paradoxes were sometimes accepted as truth instead of challenges to find the truth.

IV

Now turn to the popularity of mathematics in the general magazines. For twenty-five years the *Universal* kept its

[27] *New Monthly Magazine*, II (October 1814), 230.

[28] *Ibid.*, III (March 1815), 108.

[29] *Ibid.*, (April 1815).

[30] *Ibid.*, (May 1815).

[31] *Ibid.*, (June 1815), 419.

[32] *Ibid.*, IV (December 1815), 396.

[33] *Ibid.*, VI (September 1816), 140.

[34] *Ibid.*, VII (April 1817), 210.

problem department open, and for sixteen years more acknowledged letters to the department without printing them. The greatest number of solutions acknowledged in one issue is sixty-one, in April, 1755.[35] These came from nineteen correspondents. Twenty correspondents were recognized in April, 1753,[36] though the editors suggested that not all the contributions were original.

In all, 133 contributors solved problems sent in by 112 proposers. Discounting for some known duplications and fanciful pseudonymns, it seems there were about 160 regular contributors to the department. This was one for each eleven copies of average total circulation.[37]

The contributors lived in every corner of England, in India, and in America. The editors were grateful for such an important body of readers, and made frequent references to forthcoming articles of mathematical interest when the duel with France for world empire crowded out most topics except war, politics, and religion. Of the forty-two correspondents who signed their occupations, thirty-seven were teachers or students. The other five included a surveyor, a member of the Mathematical Society at Windsor, a cooper, a very Worthy Mathematical Clergyman, and a coachman.

It is likely that most schoolmasters and pupils signed their true names, for the masters saw here an opportunity to get free advertising. From the time C. Eodd identified himself as a tyro at Harpswell School in May, 1749, eight others signed from the same school, including the master. Various other masters quickly dropped their classical aliases and identified themselves, their pupils, and their schools.

The rise and fall of mathematics as a topic in the *Universal* may be traced through the editors' notes to correspondents. In May, 1753, it printed six questions and the note, "We have received several other curious mathematical questions, for which we are greatly obliged to our correspondents; but have not room to insert them in this number."[38] In November, 1755, there were thirteen problems, the most in any single issue. In December, 1722, the last solution to be printed accompanied a problem on the centroid of a cone.

In September, 1773, the reason for the declining use of mathematics was made plain: "The answer of A(rithalgefluxionensis) to a mathematical question will appear in our next: And we take this opportunity to assure him that it was never our intention to accuse him of plagiarism: We only hinted that questions purloined from other performances had been imposed upon us as new compositions: We would further add, that we shall be glad of any mathematical questions or disquisitions, when they tend to elucidate the practical parts of these sciences; but such questions as are calculated merely as a trial of skill between the professors of those arts we wish to avoid: We are desirous of blending utility with entertainment."[39]

In this reply the editors closed their magazine to original work, leaving it open only for what they should consider "practical disquisitions." The promised answer was never used, but it was a long time before mathematical correspondence was completely banned. This was done in April, 1788, with the blunt remark, "Our correspondent from Oxford is informed that Mathematical Questions are inadmissible."[40]

The magazine was now forty-one years old. Twelve years later it suspended publication.

[35] *Universal Magazine of Knowledge and Pleasure*, XVI (April 1755), 169–72.

[36] *Ibid.*, XII (April 1753), 167–70.

[37] Walter Graham, *English Literary Periodicals.* (New York: Thomas Nelson and Sons, 1930), p. 189.

[38] *Universal Magazine of Knowledge and Pleasure*, XII (May 1753), 219–21.

[39] *Ibid.*, LIII (October 1773), 214.

[40] *Ibid.*, LXXXII (April 1788), 223

V

We turn next to the cordial feeling of the publishers toward mathematics. The high point of this feeling is Simpson's seven years as editor of the *Ladies' Diary*, which frequently served as a source of materials for correspondents to other magazines.

Most of the popular magazines of the century carried reviews and announcements of new books. Sometimes the reviews were quite long, as when the *Monthly Review* devoted eleven pages to Clarke's translation of Lorgna's *Summation of Infinite Converging Series with Algebraic Divisors* and the reviewer's extension, "Observations on Converging Series."[41]

Typical of reviews of foreign books not translated into English is *Exshaw's Magazine's* mention of *Manuel de Trigonometrie Pratique* par M. l'abbe De La Grive and *Traite de Calcul Integral pour Servir de Suite a l'Analyse des Infinitement Petites* par M. Bougainville le jeune.[42]

Magazines devoted strictly to reviews were usually careful not to approve poor performances. That others were not always so careful is evidenced by this quotation from a review of Frances Masere's *Appendix to Mr. Frend's Principles of Algebra:* "Both these gentlemen agree in exploding from their system all negative quantities and contend not merely for the inutility of them, but for their absurdity. The (book) displays much acute reasoning and mathematical learning."[43]

Unfavorable reviews were often brutally frank, as in the case of William Scott's *The Elements of Geometry*. The critic says, "The Editor of this performance appears to be a teacher of youth; but it would be an injustice to commend his compilation. It is seldom that teachers are even slightly qualified to explain what they pretend to know; yet they are ambitions to publish elementary works. . . . Mr. Scott adds to the number of ignorant teachers, and his publications swell the list of useless productions."[44]

Not even the best mathematicians were free from unfavorable notices. There is this from the *Analytical Review* about *Some Properties of the Sum of the Divisors of Numbers* by Edmund Waring, M.D., F.R.S.: "Dr. Waring is generally considered as one of the most profound analysts of our age, but this, as well as most of his other papers, which have appeared in the transactions are (sic) so abstruse and unimportant, that it is not easy to decypher them, or to say what purpose they are intended to answer. In the present paper, there is scarcely a single line which is not involved in algebraical symbols, except the title, so that any abstract or analysis of it would be wholly unintelligible. Till Dr. W., therefore, chuses to make himself more perspicuous and useful, we must content ourselves with barely enumerating his communications, without attempting to elucidate them. But, perhaps, the doctor, like some mathematicians of old, may wish to place his sublime science out of reach of the vulgar; and if so, he has taken the most effective means to accomplish the purpose."[45]

This review gives evidence of an effort on the part of the editors to keep their readers abreast of the newest developments in mathematics. At the same time, it indicates that already the most scholarly non-mathematicians were beginning to recognize the hopelessness of the effort. The subject was rapidly growing "out of reach of the vulgar" whether or not the mathematicians wished it to.

Most magazines were friendly to mathematics, and showed the friendship in many ways. The *Gentleman's* in November, 1738, announced payment of a prize

[41] *Monthly Review* (1749), LXIV (May 1781), 329–39.

[42] *Exshaw's Magazine* (1741), XXIII (December 1754), 652.

[43] *Monthly Magazine*, VI (December 1788, Supplement), 507.

[44] *European Magazine* (1782), III (February 1753), 54.

[45] *Analytical Review* (1788), III (February 1789), 31.

of two guineas for the solution to a problem. The *Universal*, soon after its founding,[46] printed "a dissertation on the general usefulness of mathematical learning." The *Monthly*[47] printed an anonymous letter naming the most eminent mathematicians of Scotland, their schools, the courses of study, and recent publications. The *New Monthly* maintained a regular department for proceedings of the societies. It also contains a brief note on the neglected state of mathematical learning in in Great Britain[48] and a letter on the interest of mathematical questions.[49]

The friendship of the *Gentleman's* and the *Monthly* to mathematics is especially significant, since they enjoyed the largest circulation of the time, between four and five thousand each.[50]

Frequent references testify that those who would popularize mathematics in the eighteenth century had strong friends among the publishers of the first periodicals. Sometimes these friends were mistaken; sometimes their friendship was abused; sometimes they made themselves proper instruments for the diffusion of mathematics. However, like most of the educated men of the time, the editors did not know enough mathematics to evaluate the papers submitted to them. Nor could they afford a properly qualified board of referees. They were forced to rely upon the integrity and reputation of their correspondents. They held high regard for anyone who called himself a mathematician, so long as he continued to merit that regard. It was an enviable position that mathematicians still protect.

[46] *Universal Magazine of Knowledge and Pleasure*, II (January and February 1748), 22–25; 78–80.

[47] *Monthly Magazine*, VII (July 1799), 437.

[48] *New Monthly Magazine*, V (March 1816), 142.

[49] *Ibid.*, (July 1816), 513.

[50] Graham, *op. cit.*, p. 189.

VI

In summary, there is little in the popular magazines to warrant any conclusion that the educated Englishman of the eighteenth century was much closer to the frontier of mathematics than the present day educated American is to the same frontier. If the frontier has been moved further away, that only makes the modern layman seem not so far along the road.

Editors of eighteenth century periodicals were readier to use mathematics for entertainment than editors today. It is not clear whether this was because of a paucity of materials or because they thought it would be read and understood. Perhaps both reasons entered into the decision to keep their pages open to mathematics.

Mathematicians of that time enjoyed the active friendship of the great popular magazines in a way their successors can only envy.

General Education Values of Mathematics

(*Continued from page 245*)

Annual National Conference on Higher Education, p. 11 ff.): "The greater the number of educated people within a nation, the greater the national welfare . . . . As a consequence should not education consciously become a far more potent and more important instrument for the creation of national welfare and national strength than it has ever done before?

"Under this interpretation, taxes paid for the support of schools become not expenses but investment."[7] This certainly justifies increased appropriation for education, and it justifies extending our program of teaching mathematics. The worth of human personality and the value of mathematically trained citizens justifies tremendous trouble and the necessary expense.

[7] Harold W. Stoke, "Education as National Policy," (Report of the 5th Annual National Conference on Higher Education, p. 12–13). See footnote 2.

Math Around Us

By THE MATHEMATICS DEPARTMENT
Deering High School, Portland, Maine

Dave and Bill, facing audience, are doing their trigonometry homework at a library table at the left of center. Bill uses a ruler and compasses occasionally; Dave, a slide rule. They discuss their work with each other once or twice with apparent enjoyment and satisfaction.

The Moderator (a student with mature voice) is at the microphone on the left. There should be two boys with contrasting voices and a girl at the microphone on the right. The tableaux form back of a scrim. The lines should be given slowly, with strong accents, so that the audience goes along with the rhythm.

MODERATOR: Who are these boys? And what do they study?
1ST BOY: The blonde one's Bill. That's Dave, his buddy.
They're doing trigonometry. They're both math whizzes.
They always get A's in their tests and quizzes.

MODERATOR: And now tell me something about your math,
I didn't go far enough along that path.
My check-book won't balance and let me relax,
And I have an awful time with my in-come tax.

GIRL: I take geometry. We use straight lines,
Circles and squares and regular designs.
There's geometry in nature, there's geometry in art,
And even in photography it plays a big part.

2ND BOY: I like algebra. We solve equations.
We work out problems for various occasions.
I try to find x in the manner I should.
When it checks, oh, brother!—do I feel good!

1ST BOY: I think geometry is much more fun.
You prove a proposition and then you're done.
They're all so logical, no two of them the same—
You just have to follow the rules of the game.

MODERATOR: I've heard that mathematics is very, very old.
The ancient Greeks contributed a lot, I'm told.
GIRL: The Hindus and the Arabs advanced it quite a ways,
But it started 'way back in Egyptian days.

Tableau 1 (*center*)

Three Egyptians in colorful costumes stake out the vertices of a 3-4-5 right triangle of rope tied with knots. In background is a painting, sketch, or model of the Great Pyramid.

2ND BOY: They had a lot of trouble with the old river Nile.
It overflowed its banks every once-in-a-while.
They had to have surveyors reapportion the land,
Rope-stretchers were the men who gave a helping hand.

1st Boy: The triangle they're using is a 3–4–5.
It has a right angle, as sure as I'm alive.
The square of the hypotenuse alone pro-vides
For the sum of the squares of the other two sides.

Moderator: The Egyptians knew that?

Girl: Oh, yes, they did.
And they also constructed the Great Pyr-a-mid.
They really had a lot of engineering skill,
These an-cient counterparts of our Dave and Bill.

Tableau fades

Moderator: Now let's come back to the present. You know
Mathematical machines have stolen the show.
I doubt if human beings, in the work that remains,
Can hope to compete with mechanical brains.

2nd Boy: The computers, it's true, can reckon with ease
The value of π out as far as you please.
Two thousand places in the flicker of an eye—

Moderator: That sounds to me like an awful lot of pie!

1st Boy: Electronic computers are a boon to mankind,
But they'll never be the master of the hu-man mind.
In their every operation they obey man's will,
So we'll always need boys like our own Dave and Bill.

Tableau 2 (*center*)

There is a surveyor using a transit. An engineer is working on a blueprint. A Naval Officer is using a sextant. In the background is a painting or sketch of a bridge, a plane, or a streamlined train.

Girl: In peace we need the architect, the civil engineer.
In war we need the navigator and the bombardier.
Think of all the modern uses of the primitive wheel.
There's math in the gears of your automobile.

1st Boy: Mighty steel bridges and super-highways,
Vigilant radar protecting the skyways.
All sorts of instruments of delicate precision.
There's math in your radio and your television.

Spotlight gradually increases, directed on "Mathematics" portrayed as a mathematically spangled imposing figure, in rear stage center. Her gown is decorated with geometric figures cut from gilt and silver paper, etc.

Unison: (*Slower tempo for emphasis*)
Mathematics is "the queen and the servant of science,"
Friend of the common man as well as mental giants.
Her importance increases and she's won a lasting place
In the history of our culture and the progress of our race.

Tableau fades

The President's Page

COMMITTEES AND APPOINTMENTS

THE SUCCESS of a professional organization like the National Council of Teachers of Mathematics depends in large part on the work of committees and members appointed to carry out special responsibilities. This month the official list of National Council committees and appointments is given to provide the membership information on the planning activities of our organization and on the names of the nearly one hundred members who are giving generously of their time, and sometimes of their money, to carry forward our program.

According to the Constitution there are certain designated Standing Committees and other committees may be authorized by the Board of Directors. Committee members are appointed by the president subject to the approval of the Board. The Standing Committees are: Auditing, Budget, Nominations and Elections, Year Book Planning, and Publications of Current Interest (Supplementary Publications). The membership of all Standing Committees is on a rotation basis. The membership of certain other committees is also on a rotation basis, when in the view of the Board, the committee assignment is of a long term nature. The Board has given the President the responsibility of naming the chairmen of all committees with membership on the rotation basis. In the list below the date of termination of appointment is given by year for individual members or for the whole committee and it is understood that the term of appointment ends at the close of the Annual Meeting of the year designated.

Among the more recently appointed committees are those on Teacher Education in Mathematics, Publication for High School Students, and Publication for Arithmetic Teachers. The Committee on Teacher Education in Mathematics is jointly sponsored by the Mathematical Association of America and is intended to plan a follow-up of the Symposium held in August, 1952, and reported in the December, 1952 number of THE MATHEMATICS TEACHER on this page. The other two new committees are exploratory committees and are assigned the responsibilities of investigating the need for, possible nature of, and probable cost of periodicals, in the areas designated, to supplement the present program of publications of the National Council.

As has been stated on this page before it is hoped that many more members can be brought into active participation in the work of the National Council. Committees and appointments provide one kind of opportunity for this. Since quite a number of committee appointments, on old and new committees, will have to be made in 1953, members of the Council are urged to send to the president the names of those who are not active in National Council work and who can be recommended for this work on the basis of their special interests and experience in a particular kind of activity, their qualities of leadership, and their devotion to the goals of the National Council.

It is realized that appointment of any one person to several committees is not the most desirable practice but it will be recognized that certain of the committee responsibilities are such that members of the Board of Directors and past elected officers of the Council need to be closely associated with the committees. In many of the committees geographical representation has been an important consideration in committee appointments but, because of the difficulty of providing for committee meetings, this consideration can not always be foremost.

JOHN R. MAYOR, *President*

National Council Business

Affiliated Groups (1954): Mary C. Rogers, Westfield, N. J., Chairman; *Southeastern:* W. A. Gager, Gainesville, Fla.; *Southwestern:* Ida May Bernhard, Austin, Tex.; *Northwestern:* Donovan Johnson, Minneapolis, Minn.: *Northeastern:* Jackson Adkins, Exeter, N. H.

Auditing (1953): Daniel Lloyd, Washington D. C.; Ella Marth, Washington, D. C.

Budget: Henry Van Engen, Cedar Falls, Ia., Chairman (1953); Harry Charlesworth, Denver, Colo. (1954); Donovan Johnson, Minneapolis, Minn. (1955).

On Cooperation with N.E.A. (1954): Agnes Herbert, Baltimore, Md., Chairman; Allene Archer, Richmond, Va.; Harry Charlesworth, Denver, Colo.; James Zant, Stillwater, Okla.

Executive (1953): John R. Mayor, Madison, Wis.; Henry Van Engen, Cedar Falls, Ia.; Marie Wilcox, Indianapolis, Ind.

Membership Drive (1953): M. H. Ahrendt, Washington, D. C., Chairman; Jackson Adkins, Exeter, N. H.; Clifford Bell, Los Angeles, Calif.; Kenneth Brown, Washington, D. C.; W. A. Gager, Gainesville, Fla.; Lucy Hall, Wichita, Kans.; Elizabeth Roudebush, Seattle, Wash.; Henry Swain, Winnetka, Ill.; Henry Van Engen, Cedar Falls, Ia.

Nominations and Elections Committee (1953): Edith Woolsey, Minneapolis, Minn., Chairman; H. W. Charlesworth, Denver, Colo.; Agnes Herbert, Baltimore, Md.; Lenore John, Chicago, Ill.; F. L. Wren, Nashville, Tenn.

On Nominations for Editor of THE MATHEMATICS TEACHER **(1953):** Lenore John, Chicago, Ill., Chairman; Harold Fawcett, Columbus, O.; W. A. Gager, Gainesville, Fla.; Lucy Hall, Wichita, Kans.; Agnes Herbert, Baltimore, Md.; Ben Sueltz, Cortland, N. Y.; Marie Wilcox, Indianapolis, Ind.

Place of Meetings: Agnes Herbert, Baltimore, Md., Chairman (1955); Allene Archer, Richmond, Va. (1954); Harold Fawcett, Columbus, O. (1954); Lucy Hall, Wichita, Kans. (1953); Henry W. Syer, Boston, Mass. (1953); Edith Woolsey, Minneapolis, Minn. (1955).

On Records in the National Office (1953): Agnes Herbert, Baltimore, Md., Chairman; Truman Klein, Washington, D. C.; Francis G. Lankford, Jr., Charlottesville, Va.; Veryl Schult, Washington, D. C.; Dorothy Wilson, Washington, D. C.; Myrl Ahrendt, Washington, D. C. (Ex-Officio).

State Representatives (1954): M. H. Ahrendt, Washington, D. C., Chairman.

Meetings

Co-ordination of Programs: President and Vice-Presidents

Atlantic City (April 1953): Program: Mayor, Herbert, Sauble, Wilcox, Zant.
Local Arrangements: Mary C. Rogers, Westfield, N. J.

Miami (July 1953): Program: W. A. Gager, Gainesville, Fla.
Local Arrangements: Charlotte Carlton, Miami, Fla.

Kalamazoo (August 1953): Program: Irene Sauble, Detroit, Mich.
Local Arrangements: Charles H. Butler, Kalamazoo, Mich.

Los Angeles (December 1953): Program: Marie Wilcox, Indianapolis, Ind.
Local Arrangements: Clifford Bell, Los Angeles, Calif.

Cincinnati (April 1954): Program: President and Vice-Presidents
Local Arrangements: Mildred Keiffer, Cincinnati, O.

Publications

On Official Journal (1953): E. H. C. Hildebrandt, Evanston, Ill., Chairman; Phillip Jones, Ann Arbor, Mich.; Henry W. Syer, Boston, Mass.; Edith Woolsey, Minneapolis, Minn.

On Publication for Arithmetic Teachers (1953): Henry Van Engen, Cedar Falls, Ia., Chairman; Foster Grossnickle, Jersey City, N. J.; Ida Mae Heard, Lafayette, La.; E. H. C. Hildebrandt, Evanston, Ill.; Harold Moser, Towson, Md.; Sara A. Rhue, Madison, Wis.; Irene Sauble, Detroit, Mich.; D. Banks Wilburn, Huntington, W. Va.

On Publication for High School Students (1953) (jointly sponsored with The Mathematical Association of America): Harold D. Larsen, Albion, Mich., Chairman; K. Eileen Beckett, Lebanon, Ind.; Harold Fawcett, Columbus, O.; D. W. Hall, College Park, Md.; Margaret Joseph, Milwaukee, Wis.; H. L. Meyer, Jr., Chicago, Ill.

On Publications of Current Interest: Henry Syer, Boston, Mass., Chairman (1955); M. H. Ahrendt, Washington, D. C.; Charles Butler, Kalamazoo, Mich.; Hope Chipman, Ann Arbor, Mich.; Janet Height, Wakefield, Mass.; Donovan A. Johnson, Minneapolis, Minn.; Phillip Jones, Ann Arbor, Mich.; Margaret Joseph, Milwaukee, Wis.; Joy Mahachek, Indiana, Pa.; Ann C. Peters, Keene, N. H.; H. Vernon Price, Iowa City, Ia.; Henry Swain, Winnetka, Ill.

Yearbook Planning: F. Lynwood Wren, Nashville, Tenn., Chairman (1953); Frank G. Lankford, Jr., Charlottesville, Va. (1955); Daniel W. Snader, Urbana, Ill. (1954).

22nd Yearbook: John R. Clark, Lahaska, Pa., Chairman; John J. Kinsella, New York, N. Y.; Joy Mahachek Indiana, Pa.; Philip Peak, Bloomington, Ind.; Veryl Schult, Washington, D. C.

Other Professional Activities

Contests, Scholarships, Talent Search Committee (1953): Houston T. Karnes, Baton Rouge, La., Chairman; Walter H. Carnahan, Lafayette, Ind.; Daniel B. Lloyd, Washington, D. C.; Mary C. Rogers, Westfield, N. J.; Marie S. Wilcox, Indianapolis, Ind.

On Cooperation of Mathematics with Industry (1954): Phillip S. Jones, Ann Arbor, Mich., Chairman; Clifford Bell, Los Angeles, Calif.; Ida May Bernhard, Austin, Texas; Paul C. Clifford, Montclair, N. J.; Richard M. Johnston, Tulsa, Okla.; Donald W. Lentz, Parma, O.; Catherine A. V. Lyons, Pittsburgh, Pa.; W. W. Rankin, Durham, N. C.; Myron Rosskopf, New York, N. Y.; John Schacht, Columbus, O.

On Teacher Education in Mathematics (1954) (jointly sponsored with M.A.A.): Carroll V. Newsom, Albany, N. Y., Chairman; Burton W. Jones, Boulder, Colo.; Kenneth May, Northfield, Minn.; Robert Pingry, Urbana, Ill.; Alfred Putnam, Chicago, Ill.; Henry Van Engen, Cedar Falls, Ia.; Robert C. Yates, West Point, N. Y.

Publicity (1953): Kenneth E. Brown, Washington, D. C., Chairman; H. G. Ayre, Macomb, Ill.; M. H. Ahrendt, Washington, D. C.; Allene Archer, Richmond, Va.; Ida Mae Heard, Lafayette, La.; Mabel Simcox, Chicago, Ill.; Gilbert Ulmer, Lawrence, Kans.

Subcommittees:

JOURNALS OF EDUCATION: M. H. Ahrendt, Washington, D. C.; Ida Mae Heard, Lafayette, La.; Lesta Hoel, Portland, Ore.; Veryl Schult, Washington, D. C.; James H. Zant, Stillwater, Okla.

POPULAR MAGAZINES: Howard F. Fehr, New York, N. Y.; Maurice Hartung, Chicago, Ill.; Frank Lankford, Charlottesville, Va.; Elizabeth Roudebush, Seattle, Wash.; Ben Sueltz, Cortland, N. Y.

Research: H. Van Engen, Cedar Falls, Ia., Chairman (1955); Kenneth E. Brown, Washington, D. C. (1954); Howard F. Fehr, New York, N. Y. (1954); Ida Mae Heard, Lafayette, La. (1955); Lenore John, Chicago, Ill. (1953); F. Lynwood Wren, Nashville, Tenn. (1953).

National Council Representatives

National Committee on Teaching Mathematics (Associated with the International Mathematics Union) E. H. C. Hildebrandt, Evanston, Ill.; Henry W. Syer, Boston, Mass.

N.E.A. Regional Conferences on Teacher Education and Professional Standards

New York	January 2–3, 1953	Agnes Herbert, Baltimore, Md. John Kinsella, New York, N. Y.
Atlanta	January 9–10, 1953	Bess Patton, Atlanta, Ga.
Chicago	January 19–20, 1953	H. G. Ayre, Macomb, Ill. E. W. Hellmich, DeKalb, Ill.
Kansas City	January 23–24, 1953	Gilbert Ulmer, Lawrence, Kans. James Zant, Stillwater, Okla.
Colorado Springs	January 26–27, 1953	Burton Jones, Boulder, Colo.
San Francisco	January 30–31, 1953	G. T. Buswell, Berkeley, Calif. Arthur Hall, Menlo Park, Calif.

Annual Conference on Elementary Education (U. S. Office of Education, Washington, D. C., 1953): Edwina Deans, Arlington, Va.; Ben A. Sueltz, Cortland, N. Y.

N.E.A. Centennial Action Program Conference (Atlantic City, 1953): Agnes Herbert, Baltimore, Md.

(Other Representatives of the National Council were given on this page in the February, 1953 issue.)

—In all good thinking and feeling are to be found the three great ideas underlying both logic and mathematics; viz.: Generality; Form (something that can be handled when its type is recognized); and Variability.

J. Barzun, *Teacher in America*, p. 87

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

78. More About Big Numbers

Miscellanea 60 [THE MATHEMATICS TEACHER, XLV (Nov. 1952), p. 528–530] brought several replies which we publish here and in the next two items.

Arsenal Junior High School
PITTSBURGH 1, PA.
Nov. 11, 1952

DEAR DR. JONES:

In answer to Tom Hawk's letter, *Miscellanea 50,* it occurred to me that he might be looking for large-number names. In your answer, *Miscellanea 60,* you refer to George Soulés' number names, published in 1905. I am enclosing a copy of the "Appendix" to Edward Brooks' *The Philosophy of Arithmetic* (Lancaster, Pa.: 1880 and 1904). On page 100 of the text Brooks writes: "... the names of the periods above duodecillions have not been fully settled by usage. Prof. Henkle ... finds a law which he maintains should hold in the formation of the names of the higher periods ... from the Latin ordinal numerals." I have been able to find nothing further concerning Henkle.

Milli-millillion is $10^{3000003}$. At 10 digits to the inch, ordinary typewriter spacing, with no allowance for commas, one milli-millillion would be 4.75 miles long. Aaron Bakst, in *Mathematics—Its Magic and Mastery,* page 55, says a billion, or 10^9, "objects, if one were counted each second, could be counted in about 31.7 years provided the teller worked on a non-stop shift day and night, week day and holiday." But since one can count seconds by saying "one thousand one, one thousand two," or four syllables to the second, one could not keep up this rate in counting much past one thousand, one hundred.

Perhaps Tom would calculate how much time would be needed to calculate some of the larger numbers given by Henkle. Another thing for Tom to consider is this, how much further could he count, given endless time and leisure, before needing to invent a new combination of prefixes for these millillions?

Do you ever want new questions, or new forms of old questions for your readers and their students to puzzle over? Which would you rather have, one pound of fifty-cent pieces or two pounds of quarters? What shape is night?

Sincerely yours,
ELIZABETH F. BROWN

Henkle's Names of Periods

Millions (1), Billions (2), Trillions (3), Quadrillions (4), ..., Nonillions (9), Decillions (10), Undecillions (11), Duodecillions (12), Tertiodecillions (13), Quarto-decillions (14), ..., Nono-decillions (19), Vigillions (20), Primovigillions (21), Secundo-vigillions (22), ..., Nono-vigillions (29), Trigillions (30), ..., Quadragillions (40), ..., Quinquagillions (50), ..., Nonagillions (90), ..., Centillions, (100), Primo-centillions (101), ..., Decimo-centillions (110), Undecimo-centillions (111), Duodecimo-centillions (112), Tertio-decimo-centillions (113), ..., Vigesimo-centillions (120), Primo-vigesimo-centillions (121), ..., Trigesimo-centillions (130), ..., Quadragesimo-centillions (140), ..., Ducentillions (200), Trecentillions (300), Quadringentillions (400), Quingentillions (500), ..., Millillions (1000), ..., Centesimo-millillions (1100), Ducentesimo-millillions (1200), Trecentesimo-millillions (1300), Quadringentesimo-millillions (1400), ..., Nongentesimo-millillions (1900), Bi-millillions (2000), Tri-millillions (3000), Quadri-millillions (4000), ..., Deci-millillions (10,000), Undeci-millillions (11,000), Duodeci-millillions (12,000), ..., Vici-millillions (20,000), Semeli-vici-millillions (21,000), Bi-vici-millillions (22,000), ..., Trici-millillions (30,000), Quadragi-millillions (40,000), Quinquagi-millillions (50,000), ..., Centi-millillions (100,000), Semeli-centi-millillions (101,000), Bi-centi-millillions (102,000), ..., Ducenti-millillions (200,000), Trecenti-millillions (300,000), ..., Milli-millillions (1,000,000).

It should be observed that words ending in *o* represent numbers to be added and those ending in *i* represent multipliers. When two words end in *i* the sum of the numbers indicated is to be taken as a multiplier. In each, the last word indicates the number to be increased or multiplied.

Note that Henkle uses *vigillion* for the *vigintillion* which was cited in *Miscellanea 60.* This is symptomatic of the fact that names for these larger numbers have been so little needed that one can find few places where they have been written. In these few places there is general agreement as to how one would proceed to make use of a systematic Latin nomenclature to

extend the number names, but there is some disagreement in the details.

This may be a good place to note a little brochure, "Are You Suffering from Billion-itis?," published before the 1952 election by the Barnes-Gibson-Raymond Division of the Associated Spring Corporation, P.O. Box 555, Plymouth, Michigan. Its advertising and political undertones are not so great as to prevent its being useful for students. A few copies are still available *gratis* by writing to the company.*

79. One Man's Big Numbers

Those who find huge numbers interesting should not overlook the numerical researches of Horace Scudder Uhler, Professor of Physics (now emeritus) at Yale University. Professor Uhler has devoted much of his spare time to performing prodigious calculations, evaluating with great care many mathematical constants. His results include exact values of immense integers having hundreds of digits, and approximations to hundreds of decimal places for logarithms, reciprocals, roots and other quantities. It is the purpose of this note to indicate some of the numbers that are on record, and where they can be found. Appended is a list of some of Professor Uhler's papers, particularly those which contain extended numerical data. For convenience the titles are numbered, and will be referred to by number in the paragraphs below.

"In the year 1900 while working on π, I used $\tan^{-1} 1/5$ and prepared a nonconsecutive table of powers of 2 . . . " is a quotation from (21), an interesting note already referred to in *Miscellanea 12* [THE MATHEMATICS TEACHER, XLIII (Dec. 1950), 418–419]. Details of this use of powers of 2 appear in (25), where some really big integers are tabulated. Listed are the values of 2^n for $n=778$, 889, 971, 1000, 2000, 2222, 3000, 3889, 4001, powers with 235, 268, 293, 302, 603, 669, 904, 1171, 1205 digits respectively. Topping all of these is the 2466 digit value of 2^{8191} in (27). Scattered throughout the papers on Mersenne's numbers and on Lucas' sequences are various powers of 2. Thus the value of 2^n for $n=66$, 521, 607, 1279 can be found in (28); for 216 and 378 in (22); 229 in (20); 227 and 971 in (19); 199 in (18); 193 in (16); 157 in (15); 47, 53, 100 in (26); 13, 17, 31 in (27). These include cases where M_p is given, that is for $p=13$, 17, 31, 157, 193, 199, 227, 229, 521, 607, 1279 since $2^p=M_p+1$. Along with the last three of these there appear in (28) the exact values of the 14th and 15th prefect numbers and some recently discovered large primes.

In the course of his investigations which established the composite nature of the Mersenne numbers M_p corresponding to $p=157$, 167, 193, 199, 227, 229, Professor Uhler used the Lucasian sequence 4, 14, 194, $\cdots$, S_n, $\cdots$, for all but M_{199}. The proof for the latter was based on 3, 7, 47, $\cdots$. Details of the procedures involved in such investigations are supplied in (27) and (28), and need not concern us here. More suited to our present purpose is the consideration of the magnitude of the terms of 4, 14, 194, 37634, $\cdots$. Since

$$S_n=(S_{n-1})^2-2$$

the terms soon become enormous. In (20), which announces that M_{229} is composite, we find $S_5=1416317954$; and in (28), $S_6=2005956546822746114$. The 37 digit value of S_7 appears in (27), but the 74 digit S_8 seems never to have been mentioned in Professor Uhler's papers. Later terms are given, however, as follows: 147 digit S_9 in (18), 293 digit S_{10} in (19), 586 digit S_{11} in (27). S_{12}, S_{13}, S_{14} receive notice in both (27) and (28). To achieve real penetration of this sequence S_{226} and S_{388} are investigated in (22). A logarithmic computation with 123 significant figures by methods and data from (5) shows, among other things, that merely to express the number of digits in S_{226} requires a 68 digit integer. Similarly, a 117 digit integer

* Also readable in connection with big numbers is "Perfect Numbers" by Constance Reid in the *Scientific American*, vol. 188 (March 1953), pp. 84–86.

expresses the number of digits in S_{388}. For comparison recall that a four digit integer expresses the number of digits in 2^{8191}.

Before leaving matters related to powers of 2 we should consider Professor Uhler's contribution to the literature of Pythagorean number, presented in (3). There we find the exact values of the sides of a right triangle whose legs a and b and hypotenuse c are such that

$$a=2^{3998}(4+2^{-1997}),\ 1205 \text{ digits},$$
$$b=2^{3998}(3-2^{-1998}-2^{-3998}),\ 1204 \text{ digits},$$
$$c=2^{3998}(5+2^{-1998}+2^{-3998}),\ 1205 \text{ digits}.$$

It is evident from (25) that the numerical data necessary for evaluating these expressions was ready for application. To duplicate this on an easily managed scale we can substitute a suitably small integer for n in the following generalized forms of the above numerical expressions wherein $n=1999$.

$$a=2^{2n}(4+4\cdot 2^{-n}),$$
$$b=2^{2n}(3-2\cdot 2^{-n}-2^{-2n}),$$
$$c=2^{2n}(5+2\cdot 2^{-n}+2^{-2n}).$$

To make certain that such a triangle is a right triangle notice that

$$a=2^{2n}(4)(1+2^{-n}),\ c+b=2^{2n}(8),$$
$$c-b=2^{2n}(2+4\cdot 2^{-n}+2\cdot 2^{-2n}),$$

that is,

$$c-b=2^{2n}(2)(1+2^{-n})^2.$$

Then

$$(c+b)(c-b)=2^{4n}(16)(1+2^{-n})^2,$$

so that

$$c^2-b^2=a^2.$$

For this right triangle, then,

$$\tan \tfrac{1}{2}A=(c-b)/a=\tfrac{1}{2}(1+2^{-n}).$$

But if $a=4$, $b=3$, $c=5$, then $\tan \frac{1}{2}A=\frac{1}{2}$. Comparing $\frac{1}{2}(1+2^{-1999})$ with $\frac{1}{2}$ it becomes plain that, in accordance with deliberate design, the giant triangle *just barely* escapes being similar to the 3, 4, 5 triangle.

Along with the powers, in the front rank of generators of huge numbers are the factorials. "The first figure of 450! falls in the 1001th place to the left of the decimal point hence this number may be fancifully dubbed the Arabian Nights' Factorial". This is the opening remark in (24), which deals with 450! and 448!, mentions the value of 450!/400! and its factorization into primes, and developes some interesting elementary properties of consecutive factorials. "Weighted-mean factorial" refers to the expression $2(n-1)!$ which appears as a limit. The major results here recorded are actually $450!/10^{111}$ and $448!/10^{109}$ because the two factorials are shorn of their terminal zeros. These computed results are based upon and extend earlier work on factorials as presented in (6), (7), (8). Title (6) identifies a 24 page booklet containing a four page introduction and an 18 page table. The following gleaned from the table of values, may be of interest. With terminal zeros included let $n!$ have d digits. Corresponding values of n and d are

$$n=22,\ 23,\ 24,\ 41,\ 69,\ \ 70,\ 100,\ 200.$$
$$d=22,\ 23,\ 24,\ 50,\ 99,\ 101,\ 158,\ 375.$$

Material in the introduction may be used as the basis for an interesting discussion with a class or club which has had some experience with factorials. In (8) the functions are tabulated without the terminal zeros (say t in number) and hence are listed as values of $n!/10^t$. The values of n are the multiples of 5 from 305 to 400, yielding twenty immense numbers beginning with the 552 digit $305!/10^{75}$ and ending with the 770 digit $400!/10^{99}$.

Turning now from huge integers we glance at some highly extended approximate data. For convenience D denotes "decimal places", and S "significant figures".

The cube roots of 2 and 3 in (29) extend to more than $700D$, and those of 4 and 9 to $478D$. Exceeding even the $1300D$ in (14) is the approximate value of $\sqrt{2}$ to $1544D$ in (13).

In (4) the values of $1/n!$ from $n=1$ to $n=214$ are given to $475D$, and from $n=215$ to $n=369$ to $70S$. For $n=369$ the first of the S is in the 789th place to the

right of the decimal point. Derived numbers are some sines, cosines and exponentials.

The early paper (1) presents the value of $i^i = e^{-\frac{1}{2}\pi}$ and seven other values of $e^{k\pi}$ to 52 "certainly correct" D. These results are extended in (12) to $136S$ and amplified to include $e^{\pm k\pi}$, cosh $k\pi$, sinh $k\pi$ for 14 values of k.

Most of the data recorded in (9), (10), (11) are natural logarithms of π and of all primes to 113. The degree of approximation varies from $110D$ to $330D$. Some important common logarithms also appear, such as those of e, π, $\sqrt{2\pi}$, and of small primes. Miscellaneous items include *ln* 100! and some arctangents. In (9) there are also approximations to over $250D$ of $1/\pi$ and π^2. The computation of $1/\pi$ was not performed by Professor Uhler, however, but by J. W. Wrench, Jr., who has recently published a $328D$ approximation of Euler's Constant [*Mathematical Tables and Other Aids to Computation*. vol. 6 (Oct. 1952), p. 255]. Additional arctangents and natural and common logarithms, 214–$330D$, appear in (27); $290D$ *ln* 173 and *ln* 5709 in (23).

A concluding thought on reliability may not be amiss. We quote from (9): "Since even a single false digit in the published value of a basic constant can cause incalculable loss of time and energy, and since unfortunately the literature of large numbers abounds in erroneous figures, it seems desirable . . . to emphasize the care and precautions which were taken in the earnest endeavor to obtain reliable results". This exemplifies the spirit which pervades all of Professor Uhler's work. Using varying methods check, re-check, and once more check at every step from computation to printing until it is certain that completely trustworthy results are published.

In the AMERICAN MATHEMATICAL MONTHLY

(1) *On the Numerical Value of i^i*, XXVIII (March 1921), 114–116.

(2) *Multiplication of Large Numbers*, XXVIII, (Nov.–Dec. 1921), 447–448.

(3) *A Colossal Primitive Pythagorean Triangle*, LVII (May 1950), 331–332.

In the TRANSACTIONS OF THE CONNECTICUT ACADEMY OF ARTS AND SCIENCES

(4) *A New Table of Reciprocals of Factorials and Some Derived Numbers*, XXXII (January 1937), 381–434.

Privately published by H. S. Uhler, New Haven, Connecticut

(5) *Original Tables to 137 Decimal Places of Natural Logarithms for Factors of the Form $(1 \pm n \cdot 10^{-p})$, Enhanced by Auxiliary Tables of Logarithms of Small Integers.* (June 1942.)

(6) *Exact Values of the First 200 Factorials.* February 1944.

(7) *Exact Values of n!, n = 201 to 300.* Unpublished manuscript.

In the PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES

(8) *Twenty Exact Factorials Between 304! and 401!*, XXXIV (Aug. 1948), 407–412.

(9) *Log π and Other Basic Constants*, XXIV (January 1938), 23–30.

(10) *Recalculation and Extension of the Modulus and of the Logarithms of 2, 3, 5, 7, and 17.* XXVI (March 1940), 205–212.

(11) *Natural Logarithms of Small Prime Numbers*, XXIX (Oct.–Nov. 1943), 319–325.

(12) *Special Values of $e^{k\pi}$, Cosh $(k\pi)$ and Sinh $(k\pi)$ to 136 Figures*, XXXIII (February 1947), 34–41.

(13) *Many-Figure Approximations to $\sqrt{2}$, and Distribution of Digits in $\sqrt{2}$ and $1/\sqrt{2}$*, XXXVII (January 1951), 63–67.

(14) *Approximations Exceeding 1300 Decimals for $\sqrt{3}$, $1/\sqrt{3}$, Sin $(\pi/3)$ and Distribution of Digits in Them*, XXXVII (July 1951), 443–447.

(15) *First Proof that the Mersenne Number M_{157} Is Composite*, XXX (October 1944), 314–316.

(16) *On All of Mersenne's Numbers Particularly M_{193}*, XXXIV (March 1948), 102–103.

In the BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY

(17) *Note on the Mersenne Numbers M_{157} and M_{167}*, LII (February 1946), 178.

(18) *On Mersenne's Number M_{199} and Lucas's Sequences*, LIII (February 1947), 163–164.

(19) *On Mersenne's Number M_{227} and Cognate Data*, LIV (April 1948), 378–380.

In MATHEMATICAL TABLES AND OTHER AIDS TO COMPUTATION

(20) *A New Result Concerning a Mersenne Number*, II (April 1946), 94.

(21) *Huge Numbers*, II (January 1947), 224–225.

(22) *The Magnitude of Higher Terms of the Lucasian Sequence 4, 14, 194, · · ·*, III (April 1948), 142–143.

In Scripta Mathematica

(23) *A Mathematician's Tribute to the State of Israel*, XIV (Sept.–Dec. 1948), 281–283.
(24) *The Arabian Nights' Factorial and the Weighted-Mean Factorial*, XV (March 1949), 94–96.
(25) *Table of Exact Values of High Powers of 2*, XV (Sept.–Dec. 1949), 247–251.
(26) *Miscellaneous Hints for and Experiences in Computation*, XVI (March–June 1950), 31–42.
(27) *Many-Figure Values of the Logarithms of the Year of Destiny and Other Constants*, XVII (Sept.–Dec. 1951), 202–208.
(28) *A Brief History of the Investigations on Mersenne Numbers and the Latest Immense Primes*, XVIII (June 1952), 122–131.
(29) *Many-Figure Approximations for $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{4}$, and $\sqrt[3]{9}$ with χ^2 Data*, XVIII (June 1952), 173–176.

It may be of interest to some readers that *Scripta Mathematica* published by Yeshiva University, New York City, offers reprints of many of the articles that appear in its issues. Prices usually range from 10¢ to 35¢.

Adrian Struyk,
Clifton High School,
Clifton, N. J.

*80. Generating Certain Huge Pythagorean Triangles**

In the previous note mention was made of H. S. Uhler's colossal triangle, a Pythagorean primitive each of whose side lengths is expressed by over 1200 digits, and whose shape is just the slightest bit different from that of the 3–4–5 triangle. There arises immediately the problem of designing a huge primitive Pythagorean triangle to be "almost similar" to any given primitive Pythagorean triangle. One possible solution is outlined in an elementary way below.

In a given triangle ABC let a, b, c be relatively prime integers such that $a^2+b^2=c^2$, with a even, b and c odd. Then $c+b$ and $c-b$ are even,

$$(c+b)(c-b)=a^2,$$

and

$$\tan \tfrac{1}{2}A=(c-b)/a.$$

Suppose that sides a', b', c' of triangle $A'B'C'$ are such that

$$a'=kar,$$
$$c'+b'=k(c+b),$$
$$c'-b'=k(c-b)r^2,$$

where k and r are numbers to be determined. Then

$$(c'+b')(c'-b')=k^2(c+b)(c-b)r^2=k^2a^2r^2.$$

Hence

$$c'^2-b'^2=a'^2,$$

and $A'B'C'$ is a right triangle. Therefore also

$$\tan \tfrac{1}{2}A'=(c'-b')/a'=(c-b)r/a,$$

that is

$$\tan \tfrac{1}{2}A'=r\cdot\tan \tfrac{1}{2}A.$$

Now if triangle $A'B'C'$ is almost similar to triangle ABC then the angles A' and A are very nearly equal. Hence r must have a value very close to 1. To make this so take

$$r=1\pm\frac{1}{I},$$

where I is a large integer. Furthermore, to make sure that a', b', c' are integers take $k=I^2$ because of the expression $k(c-b)r^2$ above. Making these substitutions we find that, explicitly,

$$a'=aI(I\pm 1),$$
$$b'=bI^2-\tfrac{1}{2}(c-b)(\pm 2I+1),$$
$$c'=cI^2+\tfrac{1}{2}(c-b)(\pm 2I+1).$$

From these expressions it is plain that for a primitive triangle $A'B'C'$ the numbers I and $\frac{1}{2}(c-b)$ must be relatively prime. Since I is to be huge, however, while b and c are to be rather small, the removal of any existing common factor would nevertheless leave a giant triangle.

For a first numerical example take the 8–15–17 triangle to be the given one. Here $a=8$, $b=15$, $c=17$, $\frac{1}{2}(c-b)=1$. Let $I=10^5$. Then, taking the $-$ in $\pm$,

$$a'=\ \ 79{,}999{,}200{,}000;$$
$$b'=150{,}000{,}199{,}999;$$
$$c'=169{,}999{,}800{,}001.$$

(Continued on page 273)

* For earlier discussions of Pythagorean numbers see *Miscellanea 49*, The Mathematics Teacher, XLV (April, 1952), 269 ff. and the references cited there.

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Edited by William L. Schaaf
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Miscellanea

(Continued from page 269)

Finally, let the given triangle be 5–12–13, so that $a=12$, $b=5$, $c=13$, $\frac{1}{2}(c-b)=4$. Take $I=10^6$. After removal of the common factor 4

$$a'=3{,}000{,}003{,}000{,}000;$$
$$b'=1{,}249{,}997{,}999{,}999;$$
$$c'=3{,}250{,}002{,}000{,}001.$$

Adrian Struyk,
Clifton High School,
Clifton, N. J.

81. Correcting a Big Error

We did not deliberately make the error noted below, but we are pleased to find that *Miscellanea* has been read with thoughtful care by at least one person.

P. S. J.

1426 Twenty-First St., N.W.
Washington 6, D. C.
January 8, 1953

Phillip S. Jones
University of Michigan
Ann Arbor, Michigan

Dear Sir:

On page 529 in the Nov. 1952 issue of The Mathematics Teacher, it is stated: "The *Scientific American* calculated that it would take $1{,}809{,}250\times10^{69}$ columns of its magazine to print Miller and Wheeler's largest prime in their standard size type." This is an absurd misinterpretation of the statement made in the *Scientific American* which actually reads: "To print characters of this type size in the number represented by Miller and Wheeler's largest prime would take approximately $1{,}809{,}250\times10^{69}$ columns of *Scientific American*." A careful reading of *Scientific American's* statement indicates that the terminology "in the number represented" is meant to be synonymously equivalent to "in the quantity represented." With this in mind it becomes apparent that the statement in The Mathematics Teacher is absurd. Actually, to print Miller and Wheeler's largest prime in the *Scientific American* correct to the last digit in the decimal system would only take up two lines of space.

It may be of interest to indicate the basis of the computation made by *Scientific American*: Miller and Wheeler's largest prime is actually equal to

$72\times40\times1{,}809{,}250\times10^{69}$ (approximately)

and there are approximately 72 lines to a column and 40 characters to a line in *Scientific American*.

I hope that you will be good enough to point out the error made in The Mathematics Teacher in a future issue.

Very truly yours,
Julian H. Brown

Perhaps here is the place to risk another quotation, this time from *Mathematical Tables and Other Aids to Computation*, vol. VI (Oct., 1952), p. 256. This note in referring to $(2^{148}+1)/17=2098$, 89366, 57440, 58648, 61512, 64256, 61022, 25938, 63921, which was revealed as a prime in a letter by A. Ferrier dated July 14, 1951, says that this is "probably the last 'largest' prime to be identified by hand computing methods (primes like $2^{1279}-1$ would require more than a century of desk calculator work to establish by any known method). . . . "

DEVICES FOR A MATHEMATICS LABORATORY

Edited by Emil J. Berger
Monroe High School, St. Paul, Minnesota

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. Or if that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

Probability Boards*

Somewhere in their educational experiences high school students attain a sophisticated glibness about the normal distribution curve—usually before they meet it mathematically. They handle topics like "the law of averages" and "the curve" conversationally in a manner that bespeaks authority. Unfortunately this professed familiarity is seldom based on knowledge. However, the fragmentary information they do happen to possess can be used to advantage in motivating the development of a slightly more respectable attitude toward those topics in probability that can be handled by them.

One way of explaining the meaning of a normal distribution to high school students might be to have them record the weights of one hundred junior boys and prepare an appropriate histogram. As another suggestion they might be asked to record the heights of five hundred stalks of corn selected at random, or to make successive measurements of the length of a classroom with the same tape. By completing histograms as before and drawing smooth curves through the midpoints of the upper bases of the rectangular elements of these histograms, the students undertaking these activities should develop a sensible appreciation of what the normal distribution curve really means. The equation of this curve (more properly called the error curve, and also referred to as the probability curve) is

$$y = A \cdot e^{-bx^2},$$

where A and b are constants that can be determined from observable data.[1] This relation can actually be established mathematically, but the development is not elementary.[2]

By using a device known as Galton's Probability Board it is also possible to produce an approximation to the normal distribution curve mechanically. A device of this kind is illustrated by Kraitchik in his book *Mathematical Recreations*.[3] Figure 1 illustrates the essential features of a similar but somewhat more simplified device and indicates appropriate dimensions to use in construction.

The device consists essentially of a reservoir for holding shot, a network of brads, and a row of narrow compartments. To operate the device one must place a quantity of shot in the reservoir (at the top) and tilt the board in such a way that the shot rolls through the network of brads into the compartments along the lower edge. The tops of the piles of shot that end up in the compartments approximate the form of a normal curve.

To build the device procure a piece of

*Acknowledgement. The department editor wishes to express his gratitude to Professor Franklin Smith of St. Thomas College for the advice and timely assistance received from him in the preparation of this article.

[1] For a discussion of this fact see Lancelot Hogben, *Mathematics for the Million* (New York: W. W. Norton & Company, Inc., 1943), pp. 607–13.

[2] *Op. cit.*, pp. 650–51.

[3] Maurice Kraitchik, *Mathematical Recreations* (New York: W. W. Norton & Company, Inc., 1942), pp. 122–23.

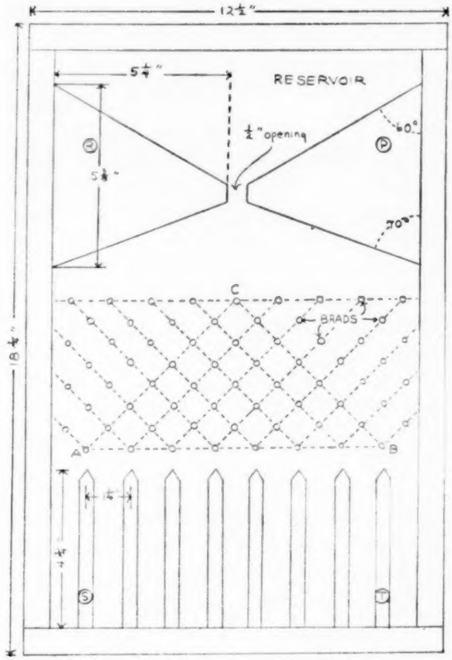

FIG. 1

$\frac{3}{8}''$ plywood $12\frac{1}{2}''\times18\frac{3}{4}''$. All other parts are to be mounted on top of this sheet of plywood including the molding along the outer edges (Fig. 2). The cross section dimensions of the molding should be $\frac{3}{4}''\times\frac{3}{4}''$.

The two trapezoidal shaped figures that form the reservoir must be $\frac{3}{8}''$ thick and should be cut in accordance with the dimensions and angular measurements that appear in the diagram. Fasten both pieces to the plywood sheet so that their uppermost points are $1''$ from the inside edge of the molding at the top.

The eight narrow strips ($\frac{3}{8}''$ wide or less) that form the compartments at the bottom of the device must also be $\frac{3}{8}''$ thick; that is, they must have the same thickness as the trapezoidal pieces. Note that the upper ends are to be bevelled. Fasten the strips to the plywood with screws or nails.

The network of brads appearing in the middle section of the diagram may be planned either as a configuration of rhombi or squares. The diagram shows the network arranged in a pattern of adjacent squares. Lines used in locating the brads have been dotted in to assist the builder with the method proposed. Note that there are exactly eight brads in the lowest row (AB)—as many as there are compartment dividers. The length of AB is $8\frac{3}{4}''$. With AB for the base complete an isosceles right triangle which will have its vertex at C. Point C should be directly below the opening in the reservoir. Draw in the parallel lines as they appear in the diagram and drive $\frac{5}{8}''$ brads down to within $\frac{3}{8}''$ of the board at all points indicated.

Suitable shot to use with the device is "steel air rifle shot." About 12 ounces are needed.

FIG. 2

As an added feature a sheet of $\frac{1}{8}''$ plastic may be fitted over the top of the device inside the framework of the molding. The exact size of this sheet should be $10\frac{7}{8}''\times17\frac{1}{8}''$. By drilling holes through it at points P, R, S, and T, the sheet may be fastened down and the device operated (and the shot returned to the reservoir) without spilling any of the shot. Glass may be substituted for the plastic sheet, but it is rather difficult to fasten and breaks rather easily.

The operation of the device was described above, but no hint was given as to the reasons why the shot arranges itself in the compartments in a form suggestinga normal distribution. Intuitively this seems like the logical thing to expect, but there is also a mathematical explanation, and fortunately one which high school students can understand—at least to a degree. However, the department editor has learned from experience that this explanation succeeds far better with the aid of a second device even though both devices work the same way and are based on the same principles. So before attempting the explanation let us consider a second "probability board."

The idea for this device came originally from the book, *Mathematical Snapshots*, by H. Steinhaus, but the dimensions and construction plans suggested here are based on a device produced by a student in the department editor's class.[4]

The main difference between the two devices is that in this second device the network of brads is replaced by a network of regular hexagonal ceramic bathroom tiles which measure 1″ between opposite sides and are $\frac{1}{4}''$ thick. In all, fifty-four pieces are needed to produce the network of tiles and alleys. Because the boy who produced this device had considerable difficulty locating a source of supply for the tiles we include here the name of one supplier (without prejudice to others) who has agreed to fill mail orders. Simply write to Mr. J. E. Wallner, Drake Marble Company, 60 Plato Street, St. Paul, Minnesota; ask for fifty-four 1″ white hexagon ceramic tiles; enclose $1.00, and you'll get the tiles.

Except that this device is larger, its construction features are similar to those of the device illustrated in Figure 1. The plywood sheet for the base has the dimensions $15\frac{5}{8}'' \times 27''$. It should be cut from material that is at least $\frac{1}{2}''$ thick in order to avoid the possibility of having it sag. Also, check to see that the sheet selected is not warped in any way. Both sag and warp will skew the curve that forms when the device is operated. Frame up the edges as before by nailing $\frac{3}{4}'' \times \frac{3}{4}''$ molding on top of the plywood sheet.

Laying out the network of tiles and alleys is not difficult but the work must be accurate; off-setting even one tile will also tend to skew the curve. Each individual tile must be oriented so that an axis of symmetry through a pair of opposite vertices is parallel to the long edges of the board. Begin by lining up the bottom row of tiles on a line 7″ from the bottom molding in such a way that there will be $\frac{3}{8}''$ between adjacent tiles and between the end pieces and the side molding. Ten pieces are needed for this first row. The $\frac{3}{8}''$ distance between tiles specified above must be observed throughout the entire pattern—that is, between all parallel sides that are next to each other. When the correct positions of all 54 tiles have been determined glue them down to the plywood with wood glue.

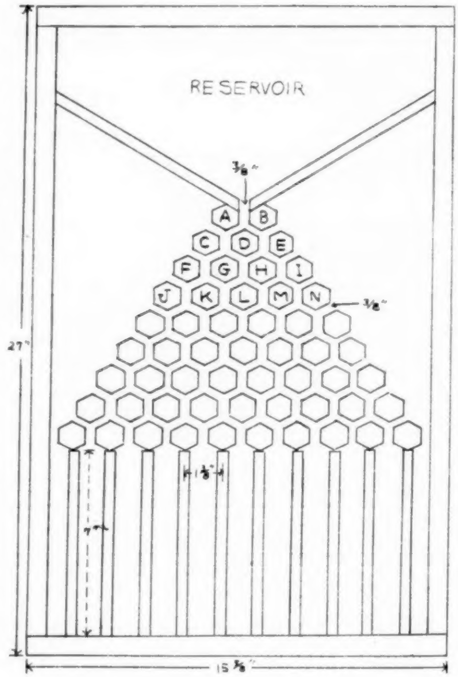

FIG. 3

The strips that form the compartment dividers at the bottom of the device should be $\frac{3}{8}''$ thick and also about that wide; their lengths for this device must be 7″. The thickness of the strips forming the reservoir should also be about $\frac{3}{8}''$; no dimension can be stated for their lengths. The builder will have to determine this measurement for himself after the tiles have been put in place.

To operate the device use about one pound of shot similar to the type recommended for the first device—steel air rifle shot. There is really no good way of returning the shot to the reservoir for successive trials except by dumping it in a

[4] H. Steinhaus, *Mathematical Snapshots* (New York: Oxford University Press, 1950), pp. 242–49.

box and transferring it back manually. A sheet of plastic fastened over the top of the molding will work, but this is expensive.

Now why does this device (as well as the previous one) produce the normal distribution curve? Let us tally the choices which one ball of shot starting from the reservoir must make to get into the different *vertical* alleys and eventually into the different compartments. It has only one (1) way of passing through alley *A-B*. Following this it can roll either through *C-D* or *D-E*. Thus there are two $(1+1)$ possibilities of getting through the second row. Suppose it rolls through *C-D*; then it can roll either into *F-G* or *G-H*. But suppose it rolls through *D-E*; then it can roll through *G-H* as before, or through *H-I*. Thus there are four $(1+2+1)$ different routes by which the shot can get into the alleys of the third row of tiles—one way of getting into *F-G*, two ways of getting into *G-H*, and one way of getting into *H-I*. How many ways are there of getting into *K-L*? The shot can get there by continuing from both of the two different paths by which it got into *G-H* or by approaching from *F-G*; in other words, there are three ways. Similarly, there are three ways of getting into *L-M*; and of course there is only one way of getting into either *J-K* or *M-N*. Thus the paths through the alleys of the fourth row of tiles may arise in eight $(1+3+3+1)$ different ways; and theoretically $\frac{1}{8}$ of all the shot starting from the reservoir will pass through *J-K*, $\frac{3}{8}$ of it through *K-L*, $\frac{3}{8}$ through *L-M*, and $\frac{1}{8}$ through *M-N*. Each fraction is the probability that a single ball of shot will pass through the particular alley with which it is written. Note that their sum is unity; or what is the same thing, that the probability that a single ball of shot will pass through the fourth row of alleys is certain.

Check back now and look at the numbers that have been written in parentheses. The reader will find that they are respectively the sums of the first four rows of Pascal's triangle. Suppose now that all the shot used with the device (one pound) is released from the reservoir through *A-B*. The heights of the piles that end up in the nine central compartments should be in proportion with the nine numbers of the ninth row of Pascal's triangle (1, 8, 28, 56, 70, 56, 28, 8, 1) taken in order. The reasoning to be applied is the same as the argument used in the case of the fourth row. Actually, the device has eleven compartments, but theoretically none of the shot should end up in the two outside sections. The reader may verify this for himself. Such small amounts as do enter the two outside sections are probably the result of forcing (running the shot too fast) or structural defects.

Continuing one step farther it can also be stated that the heights of the piles of shot will be in proportion with the coefficients of the binomial expansion $(R+L)^8$, where R is the probability that a ball of shot will turn right at a junction and L that it will turn left. Since $R=L=\frac{1}{2}$, it can be shown that the value of the rth term of this expansion is equal to the probability that a ball of shot will end up in the rth one of the nine central compartments. If the probabilities for the separate compartments are found as before, their sum will of course be unity.

As a matter of clarification let us consider a board with only three rows of tiles. The bottom row will have three alleys leading to three separate compartments. A ball of shot that turns right 2 times out of 2 will end up in the compartment on the right. A ball that turns left 2 times out of 2 will end up in the compartment on the left. And a ball that turns right only once upon entering 2 junctions will end up in the center compartment. The probabilities for the three separate events are respectively $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$. But these values are precisely the values of the terms of the expansion:

$$\begin{aligned}(R+L)^2 &= R^2+2RL+L^2\\ &= \tfrac{1}{4}+2\cdot\tfrac{1}{4}+\tfrac{1}{4},\end{aligned}$$

which equals unity as expected.

(*Continued on page 285*)

AIDS TO TEACHING

Edited by

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and

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BOOKLETS

B. 136—A Bibliography of Curriculum Materials

Audio-Visual Materials Consultation Bureau, Wayne University, 5451 Cass, Detroit 2, Michigan.

Booklet; $8\frac{1}{2}''$ by $11''$; 63 pages; \$1.25.

Description: This mimeographed bulletin prepared by a curriculum materials committee lists a variety of materials for the use of educators working on curriculum problems. The book list includes books on general curriculum development as well as books in the fields of arithmetic, language arts, science, art, music, health and physical education, mechanics of teaching, audio-visual education, and mental hygiene. The bibliography also lists pamphlets, teacher training films, curriculum guides and bulletins available at Wayne University. The sources of free and inexpensive materials are listed with descriptions of the materials and suggested uses.

Appraisal: This bulletin will be most useful in locating materials for the elementary teacher, or the curriculum specialists. The listing is incomplete and inadequate, at least in the field of arithmetic.

B. 137—Junior High School Mathematics

Iowa State Department of Public Instruction, Des Moines, Iowa.

Booklet; $5\frac{5}{8}''$by $8\frac{5}{8}''$; 115 pages; 1950; \$.45 (\$.40, 10 or more)

Description: This booklet is a curriculum guide prepared by a curriculum committee of mathematics teachers outlining the mathematics program for Iowa junior high schools. The committee recommends a single track, general in nature, for grades seven, eight, and nine. This program was adopted to provide functional competence in mathematics to the degree possible for each pupil and to reduce or eliminate the stigma attached to the general course. The content for each grade is organized around 3 or 4 important unifying principles such as using measurement and comparing quantities. Each of these units gives an introduction, objectives, content and suggested procedures, and evaluations of learning. The appendix includes a bibliography of books, magazines, audio-visual materials and textbook references.

Appraisal: This guide will be useful in suggesting the organization of mathematics around major topics such as comparing quantities. These unifying topics are also presented at different levels of difficulty so that there is continuity of treatment as well as the possibility of presentation according to the maturity of the student.

B. 138—Senior High School Mathematics

Iowa State Department of Public Instruction, Des Moines, Iowa.

Booklet; $5\frac{5}{8}''\times8\frac{5}{8}''$; 138 pages; 1949; \$.45 (\$.40, 10 or more)

Description: This bulletin prepared by a committee of mathematics teachers outlines a mathematics program for Iowa senior high schools. The committee recommends intermediate algebra for grade ten, a combined course in plane and solid

geometry for grade eleven, and two courses for grade twelve. The sequential program for the senior year covers fourth semester algebra and trigonometry while the second track course is consumer mathematics. Each course outline includes aims, content, suggested teaching procedures and pupil activities, evaluation of the unit, textbook references, and supplementary references. The appendix includes a bibliography of books, magazines, and audio-visual aids.

Appraisal: This guide will furnish the mathematics teacher many tips on the organization, materials, and teaching techniques appropriate for the topics of senior high school mathematics.

B. 139—Functional Mathematics in the Secondary Schools

State Department of Education, Attention: Division of Publications and Textbooks Services, Tallahassee, Florida.

Booklet; $8\frac{1}{2}''$ by $11''$; 117 pages; 1950; $1.00.

Description: This bulletin outlines a mathematics program for grades seven through twelve for the secondary schools of Florida. It was prepared by a committee of mathematics teachers working with consultants and curriculum specialists during two six-week workshops in the summers of 1948 and 1949. This curriculum guide calls for a dual track program for grades nine, ten, eleven, and twelve. It outlines a sequence of functional mathematics courses to parallel the traditional sequence of algebra, geometry, and trigonometry. These functional courses are built for above-average pupils as well as all others and are organized around 61 basic mathematics concepts. Following a grade placement chart, the course for each grade outlines content, teaching suggestions and references.

Appraisal: This bulletin is the first curriculum guide which outlines a complete dual track for grades nine through twelve. Its detailed outline of content, activities, and references will make it possible for secondary teachers to present these courses even though no textbooks following this pattern are now available. It is significant that it suggests that this program may be the most effective mathematics program for all secondary pupils regardless of ability. As a curriculum guide it limits its discussion to specific courses and thus does not treat general topics such as evaluation, drill, motivation, laboratory activities which are sometimes included in other guides.

B. 140—Course of Study in Mathematics for Secondary Schools

Commonwealth of Pennsylvania Department of Public Instruction, Education Building, Box 911, Harrisburg, Pennsylvania.

Booklet; $6''$ by $9''$; 295 pages; 1952; $1.25. (Do not send cash.)

Description: This comprehensive curriculum report was written by an extensive committee of teachers and specialists. It outlines a basic mathematics program for grades seven through twelve and a specialized mathematics program for grades nine through twelve. The course for each grade is outlined briefly with suggested topics, activities, materials, and illustrative units. These units include objectives, learning activities, materials such as films and pamphlets, unit tests, and bibliographies of sources of material and information. In addition to course outlines, this guide includes discussions on evaluation, objectives, integration of mathematics with other subject areas, provision for individual differences, methods of teaching, and the use of units and community resources.

Appraisal: This curriculum bulletin will provide the secondary mathematics teacher with a wealth of specific suggestions for units, projects and materials. Its outstanding feature is the many suggestions for a variety of ways to evaluate pupil progress. These suggestions include spe-

cific tests of performance, attitude, and personal development. Check sheets are also given for the teacher to evaluate the entire mathematics program including her own instruction. Since it is a collection of the ideas, practices and writing of a variety of teachers its presentation lacks uniformity. However, the teacher will usually prefer a practical treatment lacking unity over a smoothly-written, well-organized theoretical treatment.

B. 141—Timekeeping Through the Ages (LC 600)

U. S. Department of Commerce, National Bureau of Standards, Washington, D. C.

Pamphlet; mimeographed; 4 pages; free.

Description: This article is written specifically for school children. Its purpose is to aid school children in obtaining a clearer understanding of the different time pieces used in the world down through the ages. It describes briefly the use of the sundial, hourglass, sunglass, clepsydra, candles, and clocks to measure time.

Appraisal: A brief, matter-of-fact treatment written in language that a junior high school pupil can easily read.

B. 142—Units and Systems of Weights and Measures (LC 957)

U. S. Department of Commerce, National Bureau of Standards, Washington, D. C.

Pamphlet; mimeographed; 12 pages; free.

Description: This article confines its discussion of measurement to the measurement of length, mass and capacity. It begins by giving a brief history of units of measurement and measuring instruments. It discusses the units of the metric system but does not take sides in the controversy over the adoption of this system. The comparison of British and U.S. units shows considerable variance in the size of units having the same name. Subdivisions of units and arithmetical systems of numbers are related to the problem of units of measurement.

Appraisal: This article will furnish material to supplement a unit on approximate numbers or on measurement. It is not written in the language of the elementary school pupil but its conciseness should make it readable for high school students.

B. 143—Aviation Education Sources

U. S. Department of Commerce, Civil Aeronautics Administration, Aviation Education Division; Washington, D. C.

Booklet; 8″×10″; 19 pages; free.

Description: The purpose of this bulletin is to provide teachers with a comprehensive list of sources for free and inexpensive booklets, pictures, and informational material on aviation. The list includes airlines, aircraft manufacturers, organizations interested in promoting aviation and governmental bureaus. By means of a subject index and annotations, the general nature of the contribution of each organization has been indicated.

Appraisal: This bulletin will furnish an up-to-date listing of materials in the field of aviation material. The annotations will facilitate the selection of useful materials.

B. 144—Machining Alcoa Aluminum

Aluminum Company of America, Alcoa Building, Pittsburgh 19, Pa.

Booklet; 72 pages; free.

Description: This booklet presents information for tool designers and machinists so that proper tools, proper cutting speeds and feeds, and proper practice will be used for machining aluminum. In the discussion of tool shapes and uses, photographs, drawings, and tables of data are included that illustrate applications of mathematics.

Appraisal: This is a handsome booklet with high quality paper, spiral binding and photographic illustrations. The

amount of mathematical content is limited but will have some use in a shop mathematics course or geometry. Only six pages are devoted to advertising, the text being written without reference to the products of the publisher.

B. 145—The Day of Two Noons

Association of American Railroads, Transportation Building, Washington 6, D. C.

Booklet; 6″×9″; 14 pages; free.

Description: This booklet by Carlton J. Corliss tells the story of the adoption of Standard Time by the railroads. Prior to 1883 each community operated on the basis of true local or sun time. This variation in time from town to town was a source of considerable confusion particularly to the railroads whose efficient operation depends on accurate uniform time. In a large city such as Pittsburgh there were six different time standards in use to record the arrival and departure of trains. On November 18, 1883 at 12 noon, the railroads put into effect standard time according to five time zones. Although some people objected to reckoning time "contrary to nature," it soon became the time of the land. The day standard time went into effect was called "the day of two noons" because the eastern parts of each time zone had a noon based on local time and several minutes later there was another noon when their clocks were set back to standard time. On this day many unusual events and much discussion occurred as a result of the change in time.

Appraisal: This is a well written booklet with little advertising. It will be suitable supplementary reading for junior high school classes studying a unit on time.

CHARTS

C. 42—Table of Decimal Equivalents

The L. S. Starrett Company, Athol, Massachusetts.

Chart; B&W; 22″×36″; $.35 prepaid.

Description: The chart is divided vertically into three columns. The left and right columns contain exact decimal equivalents for all fractions from 1/64 to 1 in 64ths. The center column has pictures of four Starrett tools. The top and bottom are bound with metal.

Appraisal: This is a large and clear chart of decimal equivalents. The amount of advertising is not too objectionable.

EQUIPMENT

E. 122—The Magic Teacher Puzzle-Plans. (Extension of a previous review.)

Follett Publishing Company, 1257 S. Wabash Avenue, Chicago 5, Illinois, or 381 Fourth Avenue, New York 16, N. Y.

Puzzles; sets of 5″×8″ cards; $1.00 per set with discount for quantity orders.

Description: Five of the seven jig-saw puzzle sets now available in this series were reviewed previously (see E. 118 in this Department for November 1952): Sets NA1 and NA2 (Addition), NS1 and NS2 (Subtraction), and NRC (Number Concepts). The remaining two sets are similar in general nature, design and purpose. A brief description of each follows, but the reader is referred to the previous review for further deails common to all sets.

Set A3 (Addition) consists of 5 puzzle-cards. Four cards, each with 9 randomly arranged combinations, cover the 36 addition facts having sums of 11 through 18 (which are somewhat erroneously referred to in the published directions and instructions as "higher decade" combinations). The fifth card in this set presents randomly 21 zero combinations: the 19 basic ones involving zero, and also $\begin{array}{r} 10 \\ +\ \underline{0} \end{array}$ and its reverse, $\begin{array}{r} 0 \\ +\underline{10} \end{array}$.

Set M1 (Multiplication) also consists of 5 puzzle-cards. The first card includes the multiplication facts for 2's arranged in systematic "table" form, beginning with $1\times2=2$ and ending with $12\times2=24$. The

other 4 cards similarly cover the multiplication "tables" for 3's, 4's, 5's and 6's. Two things should be noted: (1) each "table" extends beyond what generally are considered to be the "basic facts," by including multipliers of 10, 11 and 12; and (2) reversals appear for some facts, such as 5×3 and 3×5, but not for others, such as 9×4.

Appraisal: Just one point, not fully considered in the earlier appraisal, will be emphasized here. Although these cards enable a child to find the answer for a combination "by himself," the child finds that answer by sensing *spatial* relations rather than *quantitative* relations. This fact obviously places serious limitations upon any use of the cards for numerous instructional purposes. (Reviewed by J. Fred Weaver, Boston University)

E. 123—Pla-Pak

The Playway Games, 18 Division Street, Sidney, New York.
Game; $1\frac{1}{2}'' \times 7'' \times 8''$; \$1.75.

Description: This game consists of a boxful of colored wooden objects. There are small cylinders ($\frac{1}{2}''$ in diameter$\times\frac{1}{2}''$), plinths ($1\frac{1}{2}''$ diameter$\times\frac{3}{8}''$), $1\frac{1}{2}''$ sticks and $4''$ sticks. A mimeographed page of directions suggests activities to teach color, counting, addition, multiplication, and division. All pieces are dyed attractive colors.

Appraisal: Such concrete materials are constantly used in elementary schools. This collection is colorful and sturdy.

FILMS

F. 86—Banks and Credit

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W (\$50) or Color (\$100); 400 ft., 10 min.

Description: A great deal of the work of a bank is summarized in very clear fashion. We hear about savings accounts, personal and commercial checking accounts, consumer credit, various types of checks (personal, certified, cashier and bank drafts), commercial and home loans, discount, and how banks invest their cash reserves. The graphic device of a pair of scales is used constantly to keep the assets and liabilities separate.

Appraisal: Since so many terms are covered in this film, no single one is described in great detail. This is not necessary because they are all familiar ideas and need to be introduced to the class through discussion, actual visits to banks and concrete materials in class. As review this film is excellent and certainly could be used in grades 8 through 10. The ideas are all important and clearly expressed.

F. 87—What is Business?

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W (\$50) or Color (\$100); 400 ft., 10 min.

Description: A short introduction tells of different types of business and the necessity for it. Then *production* is illustrated by agriculture, lumbering and fishing, mining and manufacturing; *distribution* by wholesaling and retailing; and *services* by communication, transportation, banking, lodging, food, entertainment, and others.

Appraisal: This could be used to stimulate discussion in junior or senior high school general mathematics. It is nothing more than an illustrated lecture and, although well illustrated, does not utilize the film medium to the best advantage.

F. 88—What is a Contract?

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W (\$50) or Color (\$100); 400 ft., 10 min.

Description: Four elements of contracts are considered: the material consent, by competent parties, of a legal bargain, for a consideration. All this is illustrated by two boys who make an oral contract to deliver advertising samples.

Appraisal: The story is interesting and

the material to be learned is well organized. However, for mathematics classes it is too technical and detailed. It is very suitable for business and law classes or for twelfth year general mathematics classes which might have time to devote to mature, detailed information.

F. 89—Decimals are Easy

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: The story is that of a boy who wants to buy a rubber boat and figures how to save enough money to do it. It covers the subjects of writing, reading, adding, subtracting, multiplying, and dividing decimals. In multiplication and division two different ways to place the decimal point are explained.

Appraisal: Some people will object to the use of "ragged" decimals in the examples, and others may quarrel with the methods taught for placing the decimal point in division. A comment that some important concepts are slighted should not be taken too seriously, for such omissions can be filled in by class work—a film should not try to do everything. In fact, this film tries to do a great deal. Is it intended for introduction or review?

F. 90—Work of the Stock Exchange

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W ($60) or Color ($120); 500 ft., 15 min.

Description: First we hear about some elements of production: land, management, labor and money. This leads to corporations and the need for money to expand. Finally, details of buying and selling stocks on the exchange are demonstrated.

Appraisal: The acting in this production is very amateurish and the subject proves to be somewhat unsuitable for film treatment. Even the technical quality of the photography is irregular. With all these shortcomings the film still serves a purpose of background in many mathematics classes; with a better film the purpose would be served even more.

F. 91—Sharing Economic Risk

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: After a boy's bike is stolen we are told about the way that ten boys could share the risk by pooling their money. This leads to the subject of insurance of various sorts: theft of property, auto insurance, storm or flood, and fire. Such concepts as these are introduced: insurable interest, life insurance (lump sum, and annuities), premiums and dividends, and endowment policies.

Appraisal: This is an excellent general background and at a level both understandable and interesting to secondary school pupils. It could and should lead to definite problems formulated by the class. It is not scientific, however, to present insurance in the one-sided manner without balancing a sales talk for insurance with some disadvantages of this form of savings.

F. 92—Your Thrift Habits

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: It is helpful to budget in order to save and a graph of your goal and the weekly savings toward it are useful. Two boys are shown discussing their budgets, and then Jack is seen to have trouble sticking to his scheme for buying a camera. Some other suggestions are made for saving: fix things to make them last, and buy good rather than shoddy merchandise at the beginning.

Appraisal: Here is an excellent and practical use of graphs. There are some illustrative techniques used in the pho-

tography also which represent amounts of money by piles, another graphical example. It is difficult, however, to keep a worthwhile film like this up to date with respect to prices. Some people may feel that the artificial technique of having a character talk out loud to show what he is thinking is overdone in this script.

F. 93—Parallel Lines

Johnson Hunt Productions, 1133 N. Highland Ave., Hollywood 38, Calif.

B&W ($45), Color ($90); 400 ft.

Description: The idea of parallel lines is introduced by showing railroad tracks and defining what is meant by parallel lines. The definition is further emphasized by using the lines formed by the outline of a room and showing that in order to be parallel, lines must be in the same plane. These parallel lines then move across space, out into the universe, retaining constant distance between them and never meeting, no matter how far extended. Applications of parallel lines are then shown in a shop where the axis of a lathe and the edge of the cutting tool are parallel. Three basic ideas are then brought out concerning the relationships of parallel lines. The first of these shows a draftsman using a T-Square to illustrate the theorem—"Two lines perpendicular to the same line and in the same plane are parallel." The relationships between studs and rafters of a building are used to illustrate the equal angles formed by a transversal and the equal lengths intercepted on a transversal by parallel lines. These ideas are further illustrated by animated drawings. A final application is appropriately one in which a girl locates the correct position for five equally spaced buttons on a blouse which she is sewing. Drawing six parallel lines on a sheet of paper and using the edge as a transversal, she marks off the place for each of the buttons. The film ends by reviewing the principal ideas and applications that were previously discussed.

Appraisal: The outstanding feature of this film is the demonstration of principles connected with parallel lines in terms of practical applications. The settings were realistic situations and yet the viewer did not become engrossed in personalities in the film. Probably the best place for use of the film would be as an introduction to parallel lines or, at least, early in the discussion of the subject. The film does not repeat the drawings of the textbook or teacher, rather it brings to the classroom a variety of situations in which parallel lines are a definite factor and which would not be readily available to the teacher. Also, applications are given which are appropriate for both boys and girls. The shop applications might have been improved by animation in order to emphasize the parallel lines involved which were largely invisible to the uninformed observer. However, the over-all rating of this film in terms of technical aspects as well as its treatment of the topic is excellent. (Reviewed by Robert Jackson, Denver, Colorado)

PICTURES

P. 12—Visualized Curriculum Series

Creative Educational Society, Mankato, Minnesota.

Pictures; $8\frac{1}{2}''\times 11''$; black and white; 1952; varying prices for individual sets, $89.50 for complete series in cabinet.

Description: This series of 914 pictures consists of sets of documentary photographs lithographed on heavy card stock. The pictures are grouped under seven basic social problems: food, shelter, clothing, transportation, communication, conservation of human resources, and conservation of natural resources. Each picture is a black and white photograph suitable for display on bulletin boards or for individual study. On the back of each picture there is a reading text to point out the major ideas shown by the picture. The pictures are arranged in a sequence and numbered according to an organized filing system so that any item can be found quickly and easily returned to its proper place in the file after use. A picture-find-

ing guide is included to make the use of the pictures easy.

The series on *food* includes 170 pictures on 121 cards arranged in logical sequence showing the need for feeding millions of people, the importance of food to health, the sources, production and distribution of food. The series on *shelter* includes 117 pictures on 77 cards illustrating many types of material used in building homes throughout the world, the people who help build our homes, ways of making interiors and landscapes attractive and some aspects of community planning. The series on *clothing* includes 139 pictures on 102 cards selected to show how throughout the world, the type of clothing worn is determined by the climate and culture of the society. The series on *transportation* includes 118 pictures on 99 cards showing modes of travel on land, in the air and by sea in relation to countries and people the world over. The series on *communication* includes 118 pictures on 93 cards giving a complete study of the telephone, telegraph, radio, television, newspapers, printing, mail. The series on *conservation of human resources* include 97 pictures on 92 cards presenting studies on safety, health, recreation, culture, education and public interest in governmental projects and procedures indicating the importance of conserving human resources in our democratic society. The series on *conservation of natural resources* includes 160 picture studies on 118 cards that grade school children can understand on soil conservation, flood control, wise use of forests, wild life and minerals. Pictures illustrating "What We Can Do About It" suggest interesting activities to be carried on.

Appraisal: This series of pictures will not only provide the busy elementary teacher with a series of well-selected pictures on commonly treated subjects but it will also suggest the kind and type of picture that the teacher and pupil should look for when studying these topics. Mounting cards to supplement the picture series are available from the publisher. The filing system will provide a way of keeping materials readily available. Although the series is primarily for social studies classes, elementary and junior high school teachers of mathematics will find the series on communication and transportation useful to illustrate practical problems in these areas.

Devices

(*Continued from page 277*)

The expansion could also have been written in the following form:

$$(R+L)^2 = {}_2C_2R^2L^0 + {}_2C_1R^1L^1 + {}_2C_0R^0L^2,$$

where the term ${}_2C_2\ R^2L^0$ may be interpreted as meaning the probability of making 2 right turns and 0 left turns in 2 times out of 2.

Suppose now that we consider a device with n compartments and that the amount of shot used is fixed. The corresponding binomial expansion will be $(R+L)^n$, the general term of which is ${}_nC_rR^rL^{n-r}$, where $r=n,\ (n-1),\ \cdots,\ 2,\ 1,\ 0$. As before the heights of the piles of shot will be proportional with the coefficients of the expansion. If the midpoints of the tops of these piles are connected with straight line segments, the broken line thus formed will (for large values of n) approximate the shape of the normal distribution curve previously mentioned. In fact, with a proper choice of x and y axes and with an appropriate scale, if we allow n to become infinite in the expansion of $(R+L)^n$ we obtain an equation of the form

$$y = A \cdot e^{-bx^2},$$

which is generally given as the normal distribution curve.[5]

E. J. B.

[5] For the argument in connection with the development of this idea see Hogben, *op. cit.*, pp. 650–51.

BOOK SECTION

Edited by JOSEPH STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

Elementary

Number Readiness for the Five and Six-Year Old Child, by Marge Page. Paper, 15+88 pages, 1952. Available from the author, New Mexico Western College, Silver City, New Mexico.

High School

Arithmetic for High Schools, by Charles H. Butler, Western Michigan College of Education. Cloth, xv+336 pages, 1953. D. C. Heath and Co., 285 Columbus Ave., Boston, Mass. $2.40.

Solid Geometry, A Clear Thinking Approach, by Leroy H. Schnell and Mildred G. Crawford. Cloth, x+198 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $2.96.

Solid Geometry, by William G. Shute, William W. Shirk, and George F. Porter, all of the Choate School. Cloth, vii+280 pages, 1953. American Book Co., 55 Fifth Ave., New York 3, N. Y. $2.48.

College

Elements of Mathematics, by Helen Murray Roberts and Doris Skillman Stockton, both of the University of Connecticut. Paper, vii+211 pages, 1953. Addison-Wesley Publishing Co., Kendall Square, Cambridge 42, Mass. $3.00.

Essential Business Mathematics (Second ed.) by Llewellyn R. Snyder, City College of San Francisco. Cloth, x+421 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $4.50.

Student's Workbook for Essential Business Mathematics (Second ed.), by Llewellyn R. Snyder. Paper, vi+158 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $2.50.

An Introduction to Mathematical Thought, by E. R. Stabler, Hofstra College. Cloth, xviii+268 pages, 1953. Addison-Wesley Publishing Co., Kendall Square, Cambridge 42, Mass. $4.50.

Numerical Analysis, by D. R. Hartree, University of Cambridge. Cloth, xiv+287 pages, 1952. Oxford University Press, 114 Fifth Ave., New York 11, N. Y. $6.00.

Elements of the Theory of Functions, by Konrad Knopp, University of Tubingen, translated by F. Bagemihl. Paper, 140 pages, 1952. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $1.25 ($2.50 clothbound).

Lectures on Cauchy's Problem in Linear Partial Differential Equations, by Jacques Hadamard. Paper, v+316 pages, 1952. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $1.70 ($3.50 clothbound).

The Theory of Electrons (Second ed.), by H. A. Lorentz. Paper, 343 pages, 1952. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $1.70 ($3.50 clothbound).

Miscellaneous

The Learning of Mathematics, Its Theory and Practice (Twenty-First Yearbook, The National Council of Teachers of Mathematics), edited by Howard F. Fehr. Cloth, ix+355 pages, 1953. The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D. C. $3.00 (members) $4.00 (others).

What is Science? by Norman Campbell. Paper, 186 pages, 1952. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $1.25 ($2.50 clothbound).

Directory of Secondary Day Schools, 1951–52, by Mabel C. Rice, Supervisory Survey Statistician, Research and Statistical Standards, Federal Security Agency. Paper, xviii+169 pages, 1952. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $1.00.

American School Curriculum (31st Yearbook, AASA) Cloth, 551 pages, 1953. American Association of School Administrators, 1201 Sixteenth Street, N.W., Washington 6, D. C. $5.00.

REVIEWS

Our Number Workshop, Grade 1, Maurice Hartung, Henry Van Engen, and Catharine Mahoney. Chicago, Scott Foresman and Company, 1952. 97 pp., $0.56.

Our Number Workshop is a paper backed workbook planned especially to accompany *Numbers We See* by the same publishers, but first grade teachers may use it to excellent advantage whether they are using *Numbers We See* or not. No reading by the child is required; the directions for each page are simple, and the child's execution of directions is also made uncomplicated. The writing of numbers is not required. However, as the work progresses, the child will learn to associate the written number

symbol with the proper number of objects. The quality of paper is such that it should withstand the use that is required of it. Color is used in a functional way as for example, the direction for page 15 says, "If pony number 2 is red, draw a line under him."

Our Number Workshop presents pictures to give experience with (1) making comparisons (few, many, tall, taller, big, little, as many as, and so on); (2) counting and positional use of number; (3) grouping and recognizing numbers to 10 (readiness for the basic facts); (4) measuring with an accepted unit of measurement; and (5) money (dime, nickel, penny, and their equivalent values). Addition and subtraction are not taught in this book, but work in rearranging groups is such that a readiness for these operations is built up. For the work in measurement, the teacher will need to provide each child with "measuring sticks"; the authors suggest that these be two inches long. The children are directed first to see whether certain objects are "as long as" the measuring stick, or longer, or shorter. Later, they are directed to see how many measuring sticks will be needed to reach from one object to another. What better introduction to the idea of "How many times is the selected unit used in a certain length"?

The introduction to counting by tens seems to me to be somewhat abrupt; it would require more direction than the authors have provided. *Numbers We See* gives a smoother approach to this question on page 56.

The plan by which children "tally" the number of *tens* on the left side of a diagram, and the number of *ones* on the right side, should pay good dividends in the learning of place value.

Through use of this book and the instructions planned around its familiar, every-day, child-like pictures, the teacher could be assured of giving children experiences with many fundamental ideas of the number system, and thus providing a readiness for number work to come in later grades.—ELINOR B. FLAGG, Illinois State Normal University, Normal, Illinois.

Making Algebra Plain, O. F. Revercomb. Wichita, Kan., The McCormick-Mather Publishing Company, 1952. 256 pp., $1.80.

This is a soft cover $8\frac{1}{2}''$ by $11''$ first-year algebra text. The format is excellent employing the two column layout. The print is large and dark providing for easy reading by the students.

The text covers the usual topics of first year algebra, starting with the nature of algebra and the fundamental operations and continuing through quadratic equations and simple trigonometric measurement.

The book is organized with certain psychological principles in mind. New algebraic experiences are built upon elements related to previous experience and understanding. It is also noteworthy that a continuing review is provided throughout the book. Wherever feasible, many general and specific examples of practical applications are utilized as a motivating technique. Emphasis is placed on acquiring certain skills by providing a very adequate number of practice problems. A separate set of achievement tests is provided for each student at no additional cost. These should prove of value for diagnostic purposes and also for purposes of evaluating the student's progress.—WILLIAM H. NAULT, W. K. Kellogg Junior High School, Battle Creek, Michigan.

New Plane Geometry. (Sixth ed.), A. M. Welchons and W. R. Krickenberger. Boston, Ginn and Co., 1952. viii+568 pp., $2.52.

This book, written for the student in a language which he can understand, has many helps on how to study geometry. Provision is made for individual differences by an abundance of original exercises grouped in three classes, Class A for everyone, Class B for good pupils, and Class C for a challenge to the brilliant students. Practical applications of geometric principles are stressed. A review of arithmetic and algebra is given immediately preceding the topic of geometry in which it is needed; thus these necessary skills are reviewed and maintained throughout the course. Much practice is given in reasoning in life situations with illustrations and problems which are within the comprehension of the pupils. Although this work is marked "Optional," it is one of the strong points of the book.

The treatment of locus is especially good. Locus is introduced informally early in the book and is brought forward from time to time, culminating in the integration of locus problems and algebraic equations. Supplementary topics treated at the end of this book are trigonometry, analytic geometry, aeronautics, artillery fire, map reading, and orthographic projection.

At the end of each chapter there are review questions, a list of important words, a summary of methods of proof, and several types of tests with the time for each test given. Eight comprehensive tests, each of a different type, at the end of the book also will prove helpful to the teacher.

The name *New Plane Geometry* is misleading, for much solid geometry is treated as an extension of plane geometry under the topic "Space Geometry." This textbook could be used in any class of plane geometry, but it is especially designed for the class which combines plane and solid geometry.—ALLENE ARCHER, Thomas Jefferson High School, Richmond, Virginia.

Brief Trigonometry, Revised (ed.), Edward A. Cameron. New York, Henry Holt and Company, 1952. vi+153 pp., $2.10.

This short trigonometry text carries out the statement of the author's aim: "The presentation of the essentials of plane trigonometry in a concise, readable manner so that the subject may be taught in the minimum time consistent with adequate understanding." The usual topics are covered though more briefly than generally found, and with fewer problems.

The tables of trigonometric functions to four

significant figures list angles at ten minute intervals, and interpolation for other minutes is taught, but no angles occur expressed in seconds. Angles are listed in the same tables in both degrees and radians, an excellent way of keeping their relationship constantly before the student. A good though brief section on vectors with appropriate problems appears early in the text. The study of graphs of the trigonometric functions is limited to the sine and cosine and thus deals only with finite values.

Although this text is intended for about thirty assignments in colleges or universities, it could be used in high schools as well. It is considerably easier than the texts I have used in my twelfth grade classes.—Ona Kraft, Collinwood High School, Cleveland, Ohio.

How to Study—How to Solve, H. M. Dadourian. Cambridge, Mass., Addison-Wesley Press, Inc., 1951. vi+121 pp. \$0.60.

This is an enlargement and extension of a pamphlet published in 1949. Part I is a sound discussion of mental attitude toward study, and how to get the most out of classroom, home study, and examinations. This is the best part of the book. Part II gives a list of fifteen general directions for attacking standard text book problems, with examples from geometry through calculus. Not all teachers will agree that this list is definitive, in fact the author himself ignores his own insistent advice on how to label a figure on the page succeeding the one where he gave it. This reviewer would like to see among other things, more emphasis on the value of a good figure drawn to scale, not only to suggest the method of proof, but also as a rough check on the answer. Part III is new to this edition. It seems to be a collection of parts of arithmetic, through calculus, not an outline or a summary, but rather things in each course that are frequently not mastered or not appreciated when first studied.

The booklet should be useful to the student who needs to improve his performance.—Anna S. Henriques, University of Utah, Salt Lake City, Utah.

Film and Education, edited by Godfrey Elliot. New York, Philosophical Library, 1948. xi +597 pp., \$7.50.

Thirty seven writers, prominent in their respective fields, have collaborated to present a comprehensive survey of the present and potential uses of the educational film. For one interested in films per se, or in the over-all audio-visual education program, this book appears especially valuable.

The book is divided into five parts: I. The Nature of the Educational Film, II. The Film in the Classroom, covering the subjects of music, art, social studies, etc., III. The Film Outside the Classroom, i.e. in religious education, industry and government, IV. The Education Film Abroad, and V. Administrative Problems and Practices, including those of public support and the establishment of film libraries.

I believe that the mathematics teacher interested in films for classroom use will welcome two short, but pertinent chapters. The first of these entitled "Applications of the Film in Mathematics" was written by Irene Sauble. It deals more with the use of concrete materials as teaching aids than with the film itself, but a helpful annotated list of fifty (50) mathematics films (grades 1–12) is given at the end of the chapter. The second deals with "The Selection and Evaluation of Films."

In general, this book contains a great deal of helpful information for any teacher and should make a valuable addition to one's reference library.—Randolph S. Gardner, New York State College for Teachers, Albany, New York.

Elementary Analytical Conics (Second ed.), J. H. Shackleton Bailey. London, Oxford University Press, 1950. 378 pp., \$1.75.

Many of our elementary textbooks on analytic geometry omit the more advanced topics in the conic sections. Here is an unusually thorough text on conics which includes these often omitted topics, develops them and provides exercises in which to apply them.

Both polar and parametric forms of the conics are introduced early and used throughout the book. The concept of pole and polar is developed not only with respect to the circle, but a formula is found for the polar of a point on the parabola, the ellipse and the hyperbola. The author concludes with a chapter on the conics studied with reference to oblique axes. The results are interesting, to say the least.

Every chapter of the book includes a set of exercises chosen from examinations given by English examining boards for their Higher Certificate papers. Answers are supplied.

I recommend this book for a teacher's own library. He will find the organization excellent, the exposition clear and the problems challenging.—Caroline A. Lester, New York State College for Teachers, Albany.

Elements of Statistics (Second ed.), Elmer B. Mode. New York, Prentice-Hall, 1951. xvi +346+23 pp., \$4.75.

This text is a revision of a first edition which appeared in 1941. It has been revised so that some of the representative ideas of modern statistics could be presented along with the standard topics of statistics.

Because the text stresses mathematical proof of some basic theorems and employs mathematical symbolism and terminology, the student who has had some work in the calculus will be more able to appreciate and use the text material.

The introductory chapter is a very good one. It contains sections on the meaning of statistics, basic ideas of sampling, logarithmic computation and theory, and explanations of the statistical notation used in the text. Other chapters deal with measures of central tendency and variability, frequency distributions, the normal curve,

curve fitting, regression and correlation, binomial, Poisson and Chi-Square distributions, and a few slightly more advanced concepts.

Included in the rather extensive tables are tables of logarithms, squares, cubes and reciprocals; ordinates of and area under the normal curve; values of Chi-Square, Fisher's t and F; conversion of r to z; percentage points of r when *rho* is zero.

This fine text should help develop on the part of the student a critical awareness of the elementary aspects of statistics.—IRWIN K. FEINSTEIN, Chicago Undergraduate Division, University of Illinois, Chicago, Illinois.

Analytic Geometry, John J. Corliss, Irwin K. Feinstein and Howard S. Levin. New York, Harper and Brothers Publishers, 1949. xiv+370 pp. $3.25.

This text contains ten chapters on plane analytic geometry followed by six chapters on analytic geometry of three dimensions. The topics are, in general, those usually found in the standard first course on the subject.

The authors have been careful to point out the two fundamental problems of analytic geometry and to keep them in evidence throughout the text. The geometry of the straight line receives more attention than is given in some texts. It is first developed from the standpoint of slope, then from the standpoint of direction angles. The use of direction angles of a line in two dimensions makes an easy and natural transition to the study of the properties of a line in three dimensions. Some time might be saved by omitting some of the material on the line in two dimensions from the standpoint of slope.

Unusual care has been exercised in dealing with directed segments. Although a difficulty in printing is involved, direct segments are indicated by an arrowhead throughout the book. The use of parametric equations and system of equations is a noteworthy feature. This technique provides a generalization otherwise difficult to realization.

One hundred twenty-eight pages are devoted to the treatment of conics which includes transformation of coordinates and a study of the general equation of the second degree. In addition there are some twenty-five pages devoted to the polar form of the conics. The general definition in terms of eccentricity is presented first, then the various types of conics defined as special cases. The treatment is quite thorough and includes diameters, poles, polars and a short discussion of the principle of duality. Some teachers would question the advisability of taking the conics through the routine of translation and rotation and the study of the many properties of latus rectum, tangents, diameters, etc. However this seems to be a matter of choice.

There is a good treatment of curve tracing in both rectangular and polar coordinates and the usual topics on higher plane curves. The supply of exercises is generous with numerous applied problems. The amount of material in the text is adequate and inclusive enough for a thorough first course in analytic geometry. A shorter course could be arranged by proper selection and omission. The text should serve well for students expecting to major in mathematics and also for those headed toward engineering.—H. GLENN AYRE, Western Illinois State College, Macomb, Illinois.

An Outline of Analytic Geometry, C. O. Oakley. New York, Barnes and Noble, Inc., 1950. xviii+246 pp. $1.25.

The book is a member of the College Outline Series and attempts to survey all the material covered in almost all standard text books in analytic geometry. In fact it contains a bibliography of these texts and a well-done table giving page references to each text for the many topics covered. An attempt is made to include all the several approaches to the material by the various text-book authors.

The book is far more than an outline giving rather full discussions and explanations. Many proofs are included, although formal definitions are haphazard or naive. Many excellent diagrams and figures are included, and many illustrative examples are given. There are not as many exercises as in a standard text.

The principal value of this book would be for use by students who find a need for supplementary explanatory material. It is too lengthy for a quick review of analytic geometry and is not superior to the more coherent standard texts for students who wish to learn the subject without an instructor.—C. L. SEEBECK, JR., University of Alabama, University, Alabama.

Modern Business Arithmetic—A Text-Workbook for Colleges, Thomas M. Dodds and Clyde Beighey. New York, The Gregg Publishing Company, Business Education Division, McGraw-Hill Book Company, Inc., 1950. viii+280 pp., $2.20.

Modern Business Arithmetic is presented as a text for students majoring in business education who expect to use their arithmetical knowledge as a vocational skill. The stated aims of the book include giving the student reasons for various kinds of arithmetic problems, development of speed, accuracy and skills used in business offices and in other vocational courses, orderly habits of work and an understanding of business situations which will enable the student to apply his arithmetical skills to similar situations.

To accomplish these ends the authors give 120 exercises covering the usual topics of business arithmetic; that is, the fundamental operations with whole numbers, fractions and decimals, percentage, discount, profit and loss, commissions, interest, simple and compound, bank discount, depreciation, taxes, insurance, payrolls, partnerships, corporations and sinking funds. These exercises consist of a reasonable number of problems usually preceded by a brief explanation, sometimes a single sentence, and an example. Space is left to do the necessary

calculation. Each exercise has space for name, class and date and the sheets are perforated so they can be torn out and handed in.

The explanation at the beginning of each exercise is brief, concise and accurate and, together with the illustrative example, will give an intelligent student a *method* by which the problems of the exercise can be solved. However, the understanding of the business situation involved will necessitate some explanation on the part of the teacher or an assumption of business experience which the average college student at this level does not have.

In spite of the implied criticism the book should prove very useful in elementary college commerce courses. The problems are definite and fit the situations in which they are used. The solving of them with an intelligent attitude, which can be fostered by the teacher's classroom methods, will go far toward furnishing the understanding which sometimes seems neglected in the explanations given at the beginning. The printing and topography of the book is excellent. The flexible spiral-type binding and the 8″×11″ size make the book handy for use during the class as a workbook.—JAMES H. ZANT, Oklahoma A. & M. College, Stillwater, Oklahoma.

Mathematics of Investment, Paul R. Rider and Carl H. Fischer. New York, Rinehart and Company, 1951. xi+359 pages, $5.00.

This textbook treats the traditional topics of the mathematics of investment in a simple clear manner, with little emphasis on the use of formulas *per se*. The discussion of compound interest, annuities, and life insurance functions is pitched at the level of students with a minimum of mathematical training, but more advanced material is included for the better students. A number of features are worthy of special notice. The revised notation approved in 1947 by the International Congress of Actuaries is used throughout. Modern mortality tables at modern low rates of interest replace the obsolete American Experience Table. The values of S_n^{-1} are tabulated instead of the more usual a_n^{-1}, making possible a simplified treatment of general annuities. The tables of compound interest functions are arranged by interest rate rather than by function, with five functions listed on each page under each interest rate. The rates tabulated are realistically low, starting with 1/12%. There are over one thousand well-chosen problems, with answers to each odd-numbered question printed at the end of the problem. An instructor of the mathematics of investment will want to examine this book when he chooses a new text.—HAROLD D. LARSEN, Albion College, Albion, Michigan.

Econometrics, Gerhard Tintner. New York, John Wiley and Sons, Inc., 1952. xiii+370 pp., $5.75.

This book is hardly self-contained. A student of modern mathematical statistics can find here much motivation in the abundance of "practical" problems that have been theoretically formulated. The student drawn to this book primarily as an economist but with little or no background in mathematical statistics will meet many technical difficulties.

The first part of the book discusses the role of rigorous quantitative method in economics. The second part discusses and applies some of the now classical techniques of least squares and testing of linear hypotheses in the univariate and multivariate cases. Here, at times, the distinction between unknown non-random parameters and random variables becomes rather hazy. The third part of the book deals with statistical problems in time series analysis. Here again, the main motivation comes from the analysis of variance, though there are also discussions on a certain non-parametric method and on stochastic difference equations.

There is a short appendix outlining basic computations with matrices and determinants.—MEYER DWASS, Northwestern University, Evanston, Illinois.

Life Insurance Case Analysis, Methods and Materials, Henry T. Owen. New York, Prentice-Hall, Inc., 1952. vi+109 pp., $2.50.

It is the purpose of this text to serve as a guide in the planning of a life insurance program in the format of case studies. This is aptly done, and many representative cases are given representing a good cross section of the populace.

The latter portion of the book has some excellent references, including various insurance tables with explanations, and an excellent résumé of the benefits of the newest Social Security Act.—EMIL J. WALECK, Parks College, East Saint Louis, Illinois.

Business Mathematics, Walter F. Cassidy and C. Carl Robusto. New York, Prentice-Hall, Inc., 1952. vii+304+109 pp., $4.75.

This book is written so that the student who is not too adept at mathematical reasoning, and mathematical manipulations, can still obtain the essential ideas of business mathematics. The publishers state, "Requiring only high school algebra beforehand, Cassidy and Robusto's book shies away from the unnecessary reasonings, explanations and other deviations."

It is true that a minimum of mathematical symbols are presented in this book. However it should be pointed out that the unit of measure (high school algebra) used to describe the competence required to successfully carry the course is indeed vague and indefinite. It is the reviewer's opinion that many a college junior, who has been away from his high school algebra for three or four years, will encounter some mental indigestion when they see symbols such as $d_{22}\, v_{2\frac{1}{2}\%}^{24}$, combined with others like it, stretched out across the 4.5 inches of a page. This is the ever present problem confronting any author when he is writing a book on a mathematical topic for the non-mathematical student.

In order to avoid this problem, or at least alleviate the difficulties encountered, the au-

thors use the first five chapters of the text for a review of "high school" algebra. This review includes a brief discussion of such topics as logarithms, exponents and the binomial theorem as well as the usual review topics. The remaining chapters of the book cover in a very acceptable manner the usual material found in the books on mathematics of finance.

The hand of tradition that holds texts of this type to a well defined path must be iron clad indeed. It is always difficult for the reviewer to rationalize why business educators insist on using logarithms in teaching students the basic concepts of business mathematics. In general these students are not facile at using mathematical symbols and, what is even much more to the point, this is the day of well written mathematical tables, electric calculators and electronic computers. Certainly any business office interested in those topics usually covered in a business mathematics course has, at least, a book of tables. And what about the 60 day-6% method? Is it important enough to devote any time to it in these days of exact interest tables?

The reader should not draw the conclusion that the book being reviewed is the only book using archaic methods. This seems to be the practice of most, if not all, books on business mathematics.

In general Cassidy and Robusto have written a very readable book and it seems to be teachable as well. The language is brief and to the point. The treatments of each topic are adequate and are not verbose. The book in general has a neat appearance which induces the casual observer to take another look.

As is true with most books it is always possible to disagree with the author on certain minor points. Some formulas are made to appear more complex by carrying the per cent sign into the formula. It is usually thought to be more desirable to have the variable i range over the decimal equivalents of the per cent interest rather than over the per cent range itself.—H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa.

College Algebra, Ross R. Middlemiss. New York, McGraw-Hill Book Company, 1952. xix+344 pp., $3.50.

The topics covered in this book are practically the same as in other college algebra texts. The author believes that college algebra has something to contribute to the education of students in liberal arts as well as those of engineering and other sciences. The presentation of the subject matter, therefore, is such as will contribute to basic thinking habits.

A feature of the book that will interest many teachers is the daily outline of lessons. The book is written so as to fall naturally into forty-eight lessons of about the same length. For each suggested lesson the following are given: the text pages and sections covered, the topics involved, and three approximately equivalent problem sections.

The arrangement of material is good and the statements are clear and concise. The problems appear to be well selected and graded.—L. H. Whitcraft, Ball State Teachers College, Muncie, Indiana.

Theory of Matrices, Sam Perlis. Cambridge, Addison-Wesley Press, Inc., 1952. xiv+237 pp., $5.50.

This book is an extremely lucid and well written introduction to matrix theory. The advanced undergraduate or beginning graduate student should find it ideal for his needs since the author succeeds in making the more abstract parts of the theory clear by detailed discussion.

Very little background is assumed on the part of the reader, and the book should be easily within the grasp of anyone who has mastered the calculus. Those parts of Modern Algebra which are needed in matrix theory are introduced at the appropriate places. Thus, abstract fields are defined in the first chapter and there is a chapter on polynomials in one variable over an arbitrary ground field. It is to be regretted that a book as well written and complete as this one defines a vector space only as the set of n-uples of field elements and makes no mention of the abstract definition. The discussion of inner product spaces is also very limited and a brief discussion of dual spaces might have been appropriate. There are many excellent exercises throughout some of them containing material that did not find place in the main exposition. Several applications of the subject matter are discussed in detail: for example, there is a matrix treatment of systems of ordinary differential equations with constant coefficients.

After the basic definitions of matrices and vector spaces, the author discussed equivalence and rank. There follows a chapter on determinants which are defined simply as sums of products of matrix elements rather than some sort of absolute value on the ring of all matrices. The author then treats the congruence of symmetric and Hermitian matrices. Next, similarity theory is studied and the rational canonical form is obtained over an arbitrary field. There follows a discussion of characteristic roots and the diagonalization of real symmetric, Hermitian, and finally normal matrices. The final chapter deals with the equivalence of matrices and linear transformations.

This book should prove a welcome addition to the textbooks on matrix theory and should help in providing an introduction to that subject to the non-mathematician.—Alex Rosenberg, Northwestern University, Evanston, Illinois.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by

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PRACTICE IN TEACHING OF PLANE GEOMETRY

A plan to determine practices in our classes, similar to that used in this Department in the past two years, is introduced again this month. Below is a set of questions intended to determine current classroom practices in plane geometry. Any geometry teacher may cooperate in this study by writing his answers to these questions on a post card and mailing it to J. A. Brown, Wisconsin High School, Madison 6, Wisconsin. The questions may be referred to by number and letter. If your procedure is to introduce all theorems by first proving them for the class, Ia should be checked; it you introduce only a few theorems in this way, Ic should be checked. A typical set of answers for parts I and II might be: Ia, few; b, most; c, few; d, few. IIa, few; b, none; etc. A typical set of answers for part III might be: 1a, 2f, 3b, etc.

A summary of replies received by June 15 will appear in an early number of THE MATHEMATICS TEACHER. Your help will be greatly appreciated.

I. Method used by teacher in introducing theorems.
   a) Carefully develop and explain the proof.
      All __ Most __ Few __ None __
   b) Carefully develop and explain a plan of proof.
      All __ Most __ Few __ None __
   c) Use teaching aids and induction.
      All __ Most __ Few __ None __
   d) Use a genetic method, in which pupils develop proof through carefully directed questions.
      All __ Most __ Few __ None __
   e) Other method ____________________

II. Procedures required of pupils in proving theorems.
   a) Write out proof completely (to be handed in).
      All __ Most __ Few __ None __
   b) Keep notebook with all proofs.
      All __ Most __ Few __ None __
   c) Use phrases of textbook or teacher in his proof.
      All __ Most __ Few __ None __
   d) Prove theorems without help from others.
      All __ Most __ Few __ None __
   e) Construct all figures in proof.
      All __ Most __ Few __ None __
   f) Write each authority (reason) in a complete sentence.
      All __ Most __ Few __ None __
   g) Write proof of theorems in ink.
      All __ Most __ Few __ None __
   h) Other ____________________

III. Emphasis in the course.
   Rank in order of emphasis in the course, using 1 for the item which you consider of first importance, etc.
   a) Formal proof of theorems.
   b) Original exercises.
   c) Constructions with straightedge and compasses.
   d) Applications of geometry in everyday life.
   e) Use of proof in non-geometric problems.
   f) Relations between geometry and algebra.
   g) Other ____________________

REPORT OF THE INVESTIGATION CONCERNING THE MARKING OF ANSWERS TO PROBLEMS IN ELEMENTARY SCHOOL ARITHMETIC

Miss Anna M. Ullrich, Washington School, West Allis, Wisconsin reports in *The Wisconsin Teacher of Mathematics* that many inconsistencies in the labels of answers exist in keys to textbooks of arithmetic. In order to find out some principles

that she might use as a guide in grading papers, she sent a questionnaire to elementary and junior high school teachers, elementary and junior high school principals, city elementary supervisors, superintendents of small city systems, county superintendents and supervisors, county normal school instructors, instructors of mathematics in colleges and universities, as well as to authors of books on the teaching of arithmetic, of arithmetic textbooks, of standardized tests, and of articles on elementary school arithmetic. Over 275 replies were received. Some of the conclusions reached were as follows:

1. 79% would require a label of all answers.
2. 29% would omit labels including \$, ¢, °, and %.
3. 60% would omit labels for "how many."
4. 29% would omit labels for "how much, what amount, etc."
5. 26% would omit labels for "how high, far, etc."
6. 8% would omit labels for "what amount, cost, etc."

Although the survey did not supply sufficient information which could be used in preparing the proposed guide, many interesting suggestions were received.

A few samples of the questions are listed below so that you will be able to better understand the summary of percentages given. It is not possible to give the entire questionnaire here, but if anyone wishes a complete copy he may request one by writing to Miss Ullrich and enclosing ten cents to cover costs.

SAMPLE QUESTION

I. *General.* After each question please underline "yes" or "no."
   A. In general, do you think that the answer should be labeled to be considered correct?.......... Yes No
   B. Would you mark an answer correct if the child omitted any kind of label whatever including such signs as \$, ¢, °, (degrees) and %, except in a problem specifically calling for a label, as in the case of listing recipe ingredients?............ Yes No
   C. If a problem asks, "How many?" of a certain thing or unit of measure expressed in the problem, would you mark an answer correct if there were no label?...................... Yes No
      Illustration: Miss Smith has a class of 15 boys and 12 girls. *How many pupils* has she? (Answer 27)
      Illustration: Each play costume required 4 yards of material. *How many yards* are needed for 10 such costumes? (Answer 40)

II. *Miscellaneous.* Please underline whatever answers you would consider correct for each of the following problems:
   1. A certain village has a population of 1500 while the neighboring village has a population of 750. What is the combined population of the two villages?
      2,250 or 2,250 people
   2. *What is the average speed* of an airplane that travels 1,000 miles in 5 hours?
      200 or 200 miles or 200 miles per hour
   3. *What is the average speed per hour* of an airplane that travels 1,000 miles in 5 hours?
      200 or 200 miles or 200 miles per hour
   4. Mrs. Jones used 8 eggs in a cake. *What part of a dozen* is this?
      2/3 or 2/3 dozen

No doubt many of you have some definite scheme that you follow in your own classes. Won't you share it with our readers? This need not be limited to teachers at the elementary and junior high school levels.

One reaction to the questionnaire included these remarks: "I thought by having those few sample questions we ought to have some response from readers. Take that question on population—the first under II above. The word 'people' was not used in the problem so I wouldn't expect the pupil to use it in the answer. Furthermore, we talk of population all the time without a label. For example, we say Shorewood has a population of 16,000—to me the term population implies people. Anyway I just thought we ought to get some reaction, for *I* could argue on so many."

"M" Day in Gilmer County

It was "M" Day in Gilmer County, when the Mathematics Department of the Glenville High School played host to the other four county high schools. "M" Day, in this case, meant *Mathematics Day*.

The mathematics classes of Glenville had been one of the most active groups around the school all year. Such remarks as "What's going on in there?" "Oh, you're really making it practical, are you?" "My mathematics room never looked like that." and "I never knew anyone could have that much fun in an arithmetic class." were heard every day from faculty members and older students. The Glenville students had looked forward to the occasion all year and as a sort of culmination to their year's work they were eager to share their ideas and experiences.

Some days previous to the date of the occasion, colorful invitations of a mathematical motif were sent to about one hundred guests. This list of guests included ten students, along with their principal and mathematics teacher, from each of the other high schools in the county, the members of the County Board of Education and their office staff, Glenville College faculty members, several out-of-the-county people and some local citizens who were especially interested in mathematics.

Upon arrival, the guests were greeted by two students, who supplied each of them with a packet of mathematical material. This material included crossword puzzles, magic squares, scales for adding and subtracting signed numbers, and many other interesting things that were manipulated, solved, or analyzed by each individual during the program. Interest was high on the part of everyone at all times.

With Haymen Boggs, Jr., as master of ceremonies, the group was thoroughly convinced that mathematics is not "all work and no play"—as it is often thought to be.

The program opened with the group singing *John Brown's Sister Couldn't Even Find a Sum*, followed by other mathematical songs. Fifteen students demonstrated many teaching aids or devices, all of which were inexpensive or homemade. Among these aids were a revolving angle board, showing the angles in their different positions, a triangle board, with protractors glued on, showing that the sum of the angles of any triangle equals 180°, and an angle bisector made of strips of masonite, made and demonstrated by a ninth grade student; an altitude board showing how the altitude of different kinds of triangles changes in regard to its position—often falling outside the triangle,—and aids showing the nature of and degrees in inscribed angles, central angles, and vertical angles.

Two different methods of proof of the Pythagorean theorem were demonstrated. The story of Pythagoras was given and also an extension of his theorem was presented "by proving that the area of a semi-circle drawn on the hypotenuse of a right triangle is equal to the sum of the areas of semi-circles drawn on the two legs."

One of the girls showed some special techniques in the use of a blackboard compass and showed clearly that it had its place in art also. A seventh-grade student received a great deal of praise as she drew on the board the various kinds of angles, then with free hand drawing developed these angles into about everything from a butterfly to a Red Terror Indian—the school's emblem.

Proving that mathematics can be correlated with almost any other subject, another girl, who is studying music too, seemed to "steal the show" as she explained how mathematics had helped her in her study of music, —then very beautifully sang *The Loveliest Night of the Year*.

Perhaps the most effective demonstration was done with Groves Moto-Math Set. The derivation of many of our mathematical formulas was shown and many of the students saw for the first time just

where these rules came from, and these then took on their proper meaning.

Not failing to forcefully exhibit the fact that mathematics and home economics are closely related, the guests were invited to the dining room where refreshments were served. The cookies, cut in different geometric figures, were baked by the girls as a lesson in ratio, which involved enlarging a recipe. The centerpiece for the table was made by arranging different colored drinking straws (small cylinders) into a circular base of modeling clay.

The exhibit that attracted most attention was the one on Thrift. With full cooperation on the part of the cashiers of the two local banks, it was possible to show all kinds of bank forms, notes, checks, deposit slips, savings bonds, savings stamps, and pass books. All of these had as their background toy money, both paper and silver. In the foreground was a toy adding machine, a cash register, and a collection of "piggy" banks. Other exhibits included snow flakes, flying saucers, scale drawings, straight line stitching, graphs, algebraic charts, geometric models made from paper, wire coat hangers, pipe stem cleaners and drinking straws. There was also a large collection of free material dealing with mathematics, puzzles, games, and recreations.

Much pride was also exhibited in the display of the books on mathematics that had recently been added to the library. This collection included such books as *Men of Mathematics* by E. T. Bell, *A Boy's Own Arithmetic* by Meeks, *Take a Number* by Lieber, and *Mathematics for the Million* by Hogben.

The afternoon was enjoyed, and not one present failed to agree that mathematics can be enriched and that *it really can be fun*. Perhaps the greatest compliment that came to us was from Roland Butcher, County Superintendent of Schools, who said, "I have never attended a more interesting or a more instructional program. I leave this meeting with a feeling that the teachers, the students, and the visitors are sharing in the feeling and the belief that mathematics and a good relationship among schools has been promoted."

Muriel G. Currey, President
West Virginia Council of
Mathematics Teachers
Glenville, West Virginia

The Trigonometry Class—First Assignment

"Did you know that two triangles can have five of their parts equal and the sixth parts unequal?" This question, parenthetically inserted in a conversation, was asked by a colleague. It led to a technique I like to use at the first meeting of a class in trigonometry. Here it is for your consideration.

One evening I started playing with the question, trial-and-error style at first, then abandoning the time-consuming method for a logical one. In the trial-and-error method I was reminded time and again of the effect on other parts of a triangle when one part is varied. And with the logical approach I realized I was drawing on all the plane geometry theorems on congruence of triangles and many of the theorems pertaining to similar triangles.

What could be a better review for a class beginning the study of trigonometry?

After disposing of the usual mechanics involved in the first meeting of a class, and after giving the class a brief orientation to trigonometry, the first assignment is made: "Bring to class two cardboard triangles, which have five parts of one triangle equal to five parts in the other, but the sixth parts unequal." The students are given five days to complete the assignment (outside the classroom) and advised to start on the problem early. They are loaned plane geometry textbooks from a discarded set, to use for reference.

On the day when the successful students present their triangles, they are asked to explain how they arrived at the solution. Some will reveal ingenious methods which would never have occurred to their in-

structor, and which must be recognized as good thinking.

The assignment holds still another virtue: it demands accurate work, either by protractor and ruler, or by compasses and straight-edge, in order that the parts which should be equal can be shown to be equal.

Whether or not the student produces the required cardboard triangles, if he attempts the solution he is benefited, because he must review the facts and relationships from plane geometry which are essential in trigonometry.

For the weary, the busy, or the televiewers, here is a fast outline of one solution: The six parts of a triangle are, of course, the three sides and the three angles. If two triangles have five of their parts equal and the sixth unequal, it is obvious that of the five equal parts, the three sides cannot be equal respectively or the triangles would be congruent, making the sixth parts also equal. Then it is the three angles which must be correspondingly equal, meaning the triangles are similar. The two sides which are equal respectively in the two triangles may not be correspondingly placed, or the triangles again would be congruent. The problem, then, is to have two similar triangles with two of their three sides equal respectively, but not correspondingly placed.

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Workshops and Institutes

(*Continued from page 252*)

commodations in the facilities of the University. Inquiries from persons interested in the Workshop will be welcomed by the Department of Mathematics of the University and should be addressed to Professor Davis P. Richardson, University of Arkansas, Fayetteville, Arkansas.

A Summer Institute for Mathematics Teachers will be held at the **University of Michigan,** August 3–14, 1953. The purpose of the institute will be to provide up-to-date information about new developments in mathematics, its applications, and its teaching. Particular emphasis will be placed on the uses of mathematics in industry (especially Michigan industry), and on the development from them of problems and illustrative materials, teaching aids and classroom methods which will help vitalize secondary mathematics instruction. Special lectures will be held on such topics as: Fundamentals of Quality Control in Industry; Modern Developments in Computation; Modern Computation Facilities and Their Implications for Industry and Education; Mathematics in the Machine Shop; Mathematics and Art; New Needs and Developments in Air Navigation; The Role of Mathematical Models in an Empirical Science; Mathematics in Optics and Photography; Mathematics in the Paper Industry; and Mathematics in Agriculture. The program also includes participation in laboratory work groups, discussion and study groups, and field trips. Two hours of credit may be earned by qualified students who have made arrangements in advance. Further data are available from Phillip S. Jones, Mathematics Department, University of Michigan, Ann Arbor, Michigan.

Louisiana State University's Fourth Annual Mathematics Institute will be held from June 21–27. Included on the program are a geometry laboratory and discussion groups in algebra, geometry, arithmetic, junior high school mathematics, and enrichment materials led by experts in these areas. There will be lectures given by outstanding people in mathematics and related fields. Excellent accommodations will be provided on the campus at reasonable rates. A vacation trip to interesting Louisiana can include this week for profit and enjoyment on the beautiful Louisiana State University campus. A copy of the program and additional information may be obtained by writing Houston T. Karnes, Director, Mathematics Institute, Louisiana State University, Baton Rouge, Louisiana.

Indiana University announces its **Sixth Annual Workshop for Teachers of Mathematics,** from June 22 through July 3. During this workshop the membership will have the opportunity of hearing lectures on the needs for mathematics from the fields of business, industry and science, as well as participating in groups studying problems pertaining to the teaching of mathematics in grades 7 through 14. Time and instruction in the field of laboratory materials for mathematics will also be provided. Two and one-half hours of university graduate credit will be given to those who desire it. The director of the workshop will be Walter Gingery, Visiting Instructor in Education, and Mathematics Teacher in the University School, Bloomington, Indiana. For further information please write to Philip Peak, Assistant Director, Workshop for Teachers of Mathematics, School of Education, Indiana University, Bloomington, Indiana.

Affiliated Group Activities

By IDA MAY BERNHARD, *Regional Representative, Southwestern States*
Texas Education Agency, Austin, Texas

The Ford Experiment in Arkansas

IN APRIL, 1952, the Board of Directors of the Ford Foundation's Fund for the Advancement of Education offered to finance a five-year program of teacher education in the state of Arkansas. Several months later, Arkansas educators submitted a tentative plan for this program which was accepted by representatives of the Ford Foundation.

At the present time, fifteen colleges of the state are working on this program which is being developed as an experimental project. This project is subject to continuous review with the ultimate aim either of establishing this program as the required one for all prospective public school teachers, of abandoning it altogether, or of continuing it as a part of the state's total program of teacher education. The participating colleges have reserved the right to make changes at any time in the present four-year program, or this experimental program which may appear to be beneficial to the education of the public school teachers of the state. It is believed that at least twelve years will be required for the development of this program and for an appraisal of its results.

For the year, 1952–53, approximately $474,000 was advanced from the Ford Fund to pay the expenses which would be incurred in order that the new program could be planned and put into effect. The fifteen participating colleges received from $4,500 to $50,500 to carry on the necessary planning during this year. Additional grants exceeding $100,000 will assist these colleges in paying for libraries, materials, and equipment needed in the development of this program.

The first four years of this program is to consist of work in general basic education, specialized study, and the professional guidance and orientation of prospective teachers. Since teachers in Arkansas high schools usually teach in more than one field, the prospective high school teacher will be expected to develop at least two teaching fields. The prospective elementary teacher will be expected to take specialized work which will give her broad training in several fields.

The fifth year in this experimental program is to consist of strictly professional work which will include what has usually been covered in such courses as techniques of teaching, tests and measurements, special methods, curriculum, and student teaching. Any student who elects this five-year program will be awarded a scholarship of $750 for the fifth year and will spend about one-half of this fifth year as an interne teacher in a selected high school of the state. Each interne teacher will be under the direct supervision of a "master teacher." These "master teachers" will be carefully chosen from the public school teachers of the state and will be given special training in this type of work. At the present time, it is expected that the necessary funds for the training of these "master teachers" will be advanced from the Ford Foundation's Fund.

The State Director of this five-year experimental project is Dr. Charles M. Clarke, who is also State Director of Teacher Education and Certification of the State Department of Education.

In November, at Dr. Clarke's suggestion, a committee of four public school teachers and one college teacher was chosen in each of the subject matter areas. The first assignment for these committees is the problem of determining the nature and amount of specialized study that should be offered within the first four years of this program. The State Department of Education will use the recommendations of these committees in setting up subject matter requirements for the

certification of the teachers trained under this program. Each committee must submit a final report before September 1, 1953.

Members of the Mathematics Committee are as follows: Mary Lee Foster, Henderson State Teachers College, Arkadelphia; Christine Poindexter, Little Rock Senior High School, Little Rock; Bernice Karnes, Fayetteville Senior High School, Fayetteville; C. B. Garrison, Pine Bluff Senior High School, Pine Bluff; and G. J. Jones, Principal, Corbin High School (Colored), Pine Bluff. This committee has decided to attempt to determine what competencies in mathematics are needed by every high school teacher of mathematics, and then plan a mathematics curriculum for this experimental program which will meet the needs of these teachers. At the present time, it is believed that the report of this committee may make it necessary for each of the participating colleges to offer the same required courses in mathematics.

The Ford Experiment was discussed at the fall meeting of the Arkansas Council of Teachers of Mathematics and the Arkansas Association of College Teachers of Mathematics. As a result of these discussions, the committee feels that the new mathematics curriculum must be designed to give the prospective teacher of high school mathematics the following opportunities:

(1) To acquire a proficiency in mathematics at a higher level than he will be expected to teach.
(2) To master some mathematics which he can use to enrich his teaching.
(3) To learn the mathematics that he can utilize in his everyday life.
(4) To read and study mathematical literature which will enable him to have an intelligent understanding of the part played by mathematics in the development of our modern civilization.

The Arkansas Council of Teachers of Mathematics decided to choose the work with the mathematics committee as its major project for the year, and many members of this organization are enthusiastically assisting the committee in every possible way. The mathematics people of the state believe this Ford Program can become a great program—one which will train superior teachers for the public schools of Arkansas, and one which may serve as a model for teacher training programs in other states.

Mary Lee Foster
Henderson State Teachers College
Arkadelphia, Arkansas

The Regents of the University of Minnesota, upon unanimous recommendation of the Faculty Committee on Honors, and the Administrative Committee of the Senate, voted to present to **William David Reeve,** Professor Emeritus of Mathematics at Teachers College, Columbia University, New York City, "**The Outstanding Achievement Award** of the University, which is reserved for former students of the Institution who have attained high eminence and distinction." The award was presented on the evening of March 31, 1953, at a dinner in the Main Ballroom of the Coffman Memorial Building on the occasion of the dedication of the new College of Education Building on the University of Minnesota campus. On the evening of March 30, Professor Reeve addressed the members of the Twin City Mathematics Club on "The History and Teaching of General Mathematics in the Secondary Schools of the United States." Professor Reeve was for many years head of the department of mathematics and principal of the University of Minnesota High School until he went to Teachers College in 1923.